Warsaw University of Technology

Faculty of Power and Aeronautical Engineering
WARSAW UNIVERSITY OF TECHNOLOGY

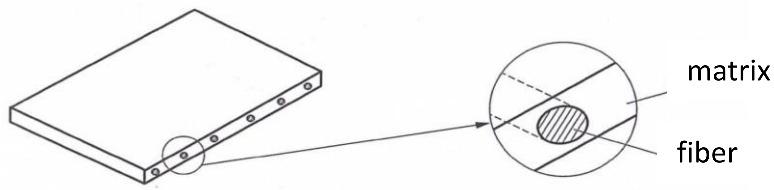
Institute of Aeronautics and Applied Mechanics

Finite element method 2 (FEM 2)

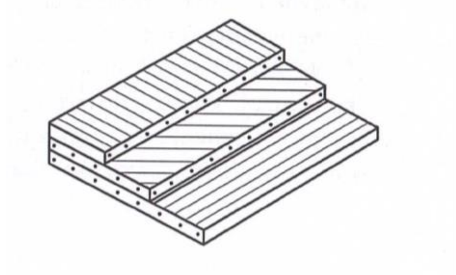
Composite structures

BASIC PROPERTIES OF COMPOSITE STRUCTURES

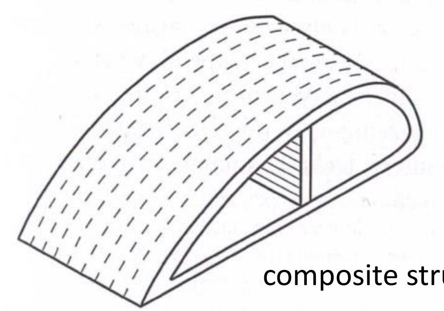
- heterogeneity
- direction-dependent material properties (anisotropy, orthotropy)
- complexity of constitutive laws
- multiscale modeling (from micro- to macroscale)
- main advantage: structures with properties optimized for given loads



single layer



multilayer composite



composite structure

Ortotropic material

Three perpendicular symmetry planes of material properties are a very common case in engineering practice. In this case, the number of independent coefficients in the constitutive matrix is reduced to 9, and the material is called orthotropic:

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_{11}} & -\frac{\nu_{21}}{E_{22}} & -\frac{\nu_{31}}{E_{33}} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_{11}} & \frac{1}{E_{22}} & -\frac{\nu_{32}}{E_{33}} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_{11}} & -\frac{\nu_{23}}{E_{22}} & \frac{1}{E_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}$$

E_{11}, E_{22} , E_{33} - Young's moduli in directions: 1, 2, 3
G_{12}, G_{23} , G_{31} - shear (Kirchoff's) moduli in planes: 12, 23 and 31
$\nu_{12}, \nu_{21}, \nu_{23}, \nu_{32}, \nu_{31}, \nu_{13}$ – Poisson's ratios

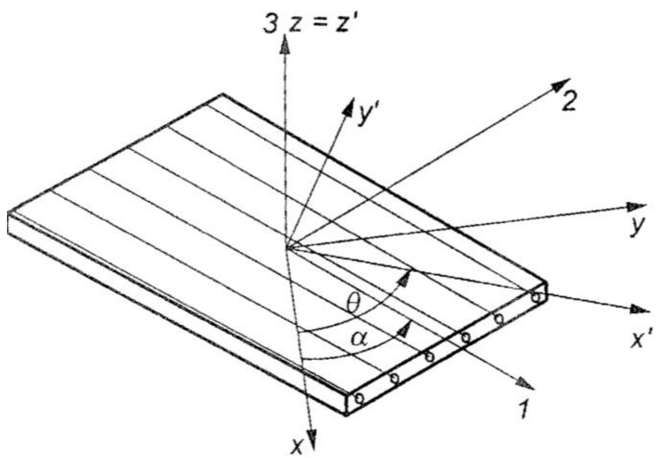
Due to symmetry:

$$\frac{v_{12}}{E_{11}} = \frac{v_{21}}{E_{22}}, \quad \frac{v_{13}}{E_{11}} = \frac{v_{31}}{E_{33}}, \quad \frac{v_{23}}{E_{22}} = \frac{v_{32}}{E_{33}}.$$

Data acquisition:

E_{11}, E_{22} , E_{33} – tensile tests along the principal orthotropy axes,
G_{12}, G_{23} , G_{31} – shear tests,
$\nu ij = -\frac{\varepsilon_{jj}}{\varepsilon_{ii}}$ – uniaxial tension tests along σ_{ii} directions.

STRAIN AND STRESS IN ORTHOTROPIC LAYER



$$\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \gamma_{23}\\ \gamma_{31}\\ \gamma_{12}\end{Bmatrix} = \begin{bmatrix} \frac{1}{E_{11}} & -\frac{\nu_{21}}{E_{22}} & -\frac{\nu_{31}}{E_{33}} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_{11}} & \frac{1}{E_{22}} & -\frac{\nu_{32}}{E_{33}} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_{11}} & -\frac{\nu_{23}}{E_{22}} & \frac{1}{E_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \tau_{23}\\ \tau_{31}\\ \tau_{12}\end{Bmatrix}$$

If a thin layer is loaded in xy-plane along the principal orthotropy direction 1 and/or 2, then:

$$\tau_{xz} = \tau_{yz} = \sigma_{zz} = 0$$

$$\tau_{31} = \tau_{23} = \sigma_{33} = 0$$

$$\boxed{\gamma_{31} = 0, \qquad \gamma_{23} = 0, \qquad \varepsilon_{33} = -\frac{\nu_{13}}{E_{11}}\sigma_{11} - \frac{\nu_{23}}{E_{22}}\sigma_{22}}$$

PLANE STRESS CONDITIONS

Constitutive law for the plane stress in 12-plane can be expressed as:

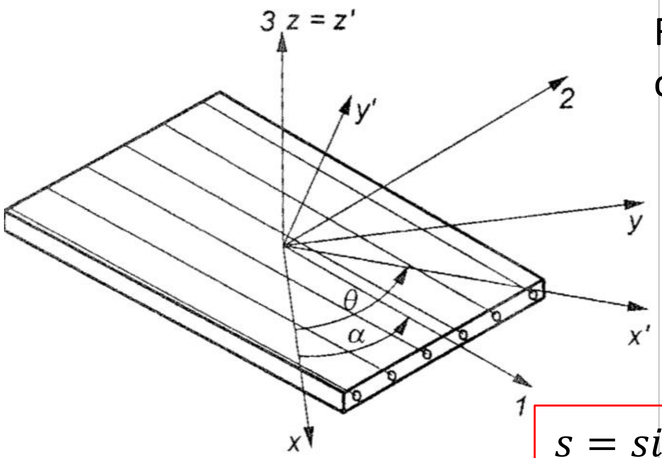
$$\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\end{Bmatrix} = \begin{bmatrix} \frac{1}{E_{11}} & -\frac{\nu_{21}}{E_{22}} & 0 \\ -\frac{\nu_{12}}{E_{11}} & \frac{1}{E_{22}} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\end{Bmatrix}$$

or:

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\end{Bmatrix} = \begin{bmatrix} \frac{E_{11}}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{21}E_{11}}{1-\nu_{12}\cdot\nu_{21}} & \frac{E_{22}}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\end{Bmatrix}$$

There are four independent constants in a constitutive matrix (E_{11}, E_{22}, ν_{12}, G_{12}).

It is usually assumed that $E_{11} > E_{22}$, ν_{12} is called major Poisson's ratio, and ν_{21} - minor Poisson's ratio.



Relationship between strain and stress components in xy- plane can be expressed as:

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} = \begin{bmatrix} S^*_{11} & S^*_{12} & S^*_{16}\\ S^*_{12} & S^*_{22} & S^*_{26}\\ S^*_{16} & S^*_{26} & S^*_{66}\end{bmatrix}\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{Bmatrix}$$

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{Bmatrix} = \begin{bmatrix} Q^*_{11} & Q^*_{12} & Q^*_{16}\\ Q^*_{12} & Q^*_{22} & Q^*_{26}\\ Q^*_{16} & Q^*_{26} & Q^*_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}$$

$$s = sin\,\alpha,\ c = cos\,\alpha$$

$$\begin{aligned}
S^*_{11} &= S_{11}c^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}s^4\\
S^*_{12} &= S_{12}c^4 + (S_{11} + S_{22} - S_{66})s^2c^2 + S_{12}s^4\\
S^*_{22} &= S_{22}c^4 + (2S_{12} + S_{66})s^2c^2 + S_{11}s^4\\
S^*_{66} &= S_{66}c^4 + 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})s^2c^2 + S_{66}s^4\\
S^*_{16} &= (2S_{11} - 2S_{12} - S_{66})sc^3 - (2S_{22} - 2S_{12} - S_{66})s^3c\\
S^*_{26} &= (2S_{12} + S_{66} - 2S_{22})sc^3 - (2S_{12} + S_{66} - 2S_{11})s^3c
\end{aligned}$$

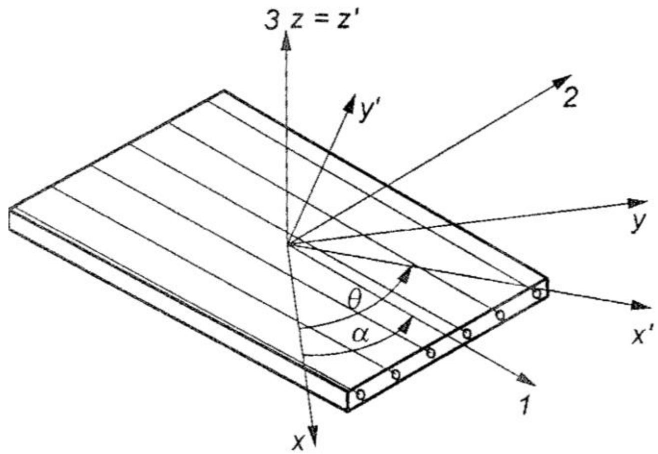
$$S_{11} = \frac{1}{E_{11}} \qquad S_{12} = \frac{-\nu_{21}}{E_{22}} \qquad S_{22} = \frac{1}{E_{22}} \qquad S_{66} = \frac{1}{G_{12}}$$

$$\begin{aligned}
Q^*_{11} &= Q_{11}c^4 + (Q_{12} + 2Q_{66})s^2c^2 + Q_{22}s^4\\
Q^*_{12} &= S_{12}c^4 + (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}s^4\\
Q^*_{22} &= Q_{22}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2 + Q_{11}s^4\\
Q^*_{66} &= Q_{66}c^4 + (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}s^4\\
Q^*_{16} &= (Q_{11} - Q_{12} - 2Q_{66})sc^3 - (Q_{12} - Q_{22} - 2Q_{66})s^3c\\
Q^*_{26} &= (Q_{12} + Q_{22} + 2Q_{66})sc^3 - (Q_{11} + Q_{12} - 2Q_{66})s^3c
\end{aligned}$$

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}} \qquad Q_{12} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}} \qquad Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}} \qquad Q_{66} = G_{12}$$

If $\alpha \neq k\frac{\pi}{2}$ the principal stress directions do not coincide with the principal strain directions (full coupling).

Strain and stress components transformation of to x'y'-coordinate system



$$\begin{Bmatrix} \sigma_{x'x'} \\ \sigma_{y'y'} \\ \tau_{x'y'} \end{Bmatrix} = [T] \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_{x'x'} \\ \varepsilon_{y'y'} \\ \dfrac{\gamma_{x'y'}}{2} \end{Bmatrix} = [T] \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \dfrac{\gamma_{xy}}{2} \end{Bmatrix}$$

Transformation matrix:

$$[T] = \begin{bmatrix} cos^2\,\theta & sin^2\,\theta & 2\,sin\,\theta\,cos\,\theta \\ sin^2\,\theta & cos^2\,\theta & -2\,sin\,\theta\,cos\,\theta \\ -\,sin\,\theta\,cos\,\theta & sin\,\theta\,cos\,\theta & cos^2\,\theta - sin^2\,\theta \end{bmatrix}$$

FAILURE CRITERIA

In the case of orthotropy, uniaxial stretching at a given load may lead to failure in some directions but not in others. This effect does not occur in isotropic materials. Therefore, orthotropic materials require different failure criteria, which are often modifications of those developed for isotropic materials.

Maximum stress criterion

The safe state of the orthotropic layer means that stresses along the principal axes of orthotropy are lower than the experimentally determined limit values X_{ti} , X_{ci} and S_{ij}^{w} .
(t- tension, c- compression, S- shear, $X_{ci} \neq -X_{ti}$).

$$X_{c1} < \sigma_{11} < X_{t2} \;\; ; \quad X_{c2} < \sigma_{22} < X_{t2} \;\; ; \quad X_{c3} < \sigma_{33} < X_{t3}$$

$$|\tau_{12}| < S^{w}{}_{12} \;\; ; \quad |\tau_{23}| < S^{w}{}_{23} \;\; ; \quad |\tau_{13}| < S^{w}{}_{31}$$

The criterion of maximum stress can be expressed as:

$$k = \max\left\{\frac{\sigma_{11}}{X_{t1}} \text{or} \frac{\sigma_{11}}{X_{c1}},\ \frac{\sigma_{22}}{X_{t2}} \text{or} \frac{\sigma_{22}}{X_{c2}},\ \frac{\sigma_{33}}{X_{t3}} \text{or} \frac{\sigma_{33}}{X_{c3}},\ \frac{|\tau_{12}|}{S_{12}^{w}},\ \frac{|\tau_{23}|}{S_{23}^{w}},\ \frac{|\tau_{13}|}{S_{31}^{w}}\right\}$$

(k= 0 – no stress, k= 1 – failure)

Maximum strain criterion

The safe state of the orthotropic layer means that strains along the principal axes of orthotropy are lower than the experimentally determined limit strain values ε_{ti} , ε_{ci} (t- tension, c- compression), and γ_{ij}^{*} - shear:

$$\boxed{\begin{array}{lll} \varepsilon_{c1} < \varepsilon_{11} < \varepsilon_{t2}\,, & \varepsilon_{c2} < \varepsilon_{22} < \varepsilon_{t2}\,, & \varepsilon_{c3} < \varepsilon_{33} < \varepsilon_{t3} \\ |\gamma_{12}| < \gamma_{12}^{*}\,, & |\gamma_{23}| < \gamma_{23}^{*}\,, & |\gamma_{31}| < \gamma_{31}^{*} \end{array}}$$

The criterion of maximum strain can be expressed as a single factor:

$$\boxed{k = \max\left\{\frac{\varepsilon_{11}}{\varepsilon_{t1}} \text{ or } \frac{\varepsilon_{11}}{\varepsilon_{c1}}, \quad \frac{\varepsilon_{22}}{\varepsilon_{t2}} \text{ or } \frac{\varepsilon_{22}}{\varepsilon_{c2}}, \quad \frac{\varepsilon_{33}}{\varepsilon_{t3}} \text{ or } \frac{\varepsilon_{33}}{\varepsilon_{c3}}, \quad \frac{|\gamma_{12}|}{\gamma_{12}^{*}}, \quad \frac{|\gamma_{23}|}{\gamma_{23}^{*}}, \quad \frac{|\gamma_{13}|}{\gamma_{13}^{*}}\right\}}$$

(k= 0 – no stress, k= 1 – failure)

Tsai–Wu criterion

This criterion includes contribution of all stress components and assumes that the material effort represents a polynomial function of stress. The safe state of the orthotropic layer is inside the ellipse:

$$F_1\sigma_{11} + F_2\sigma_{22} + F_6\tau_{12} + F_{11}\sigma_{11}^2 + F_{22}\sigma_{22}^2 + F_{66}\tau_{12}^2 + 2F_{12}\sigma_{11}\sigma_{22} = 1$$

; where material constants: $F_1, F_2, F_6, F_{11}, F_{22}, F_{66}, F_{12}$ depend on the limit values: $X_{c1}, X_{t1}, X_{c2}, X_{t2}$, $S^w{}_{12}$:

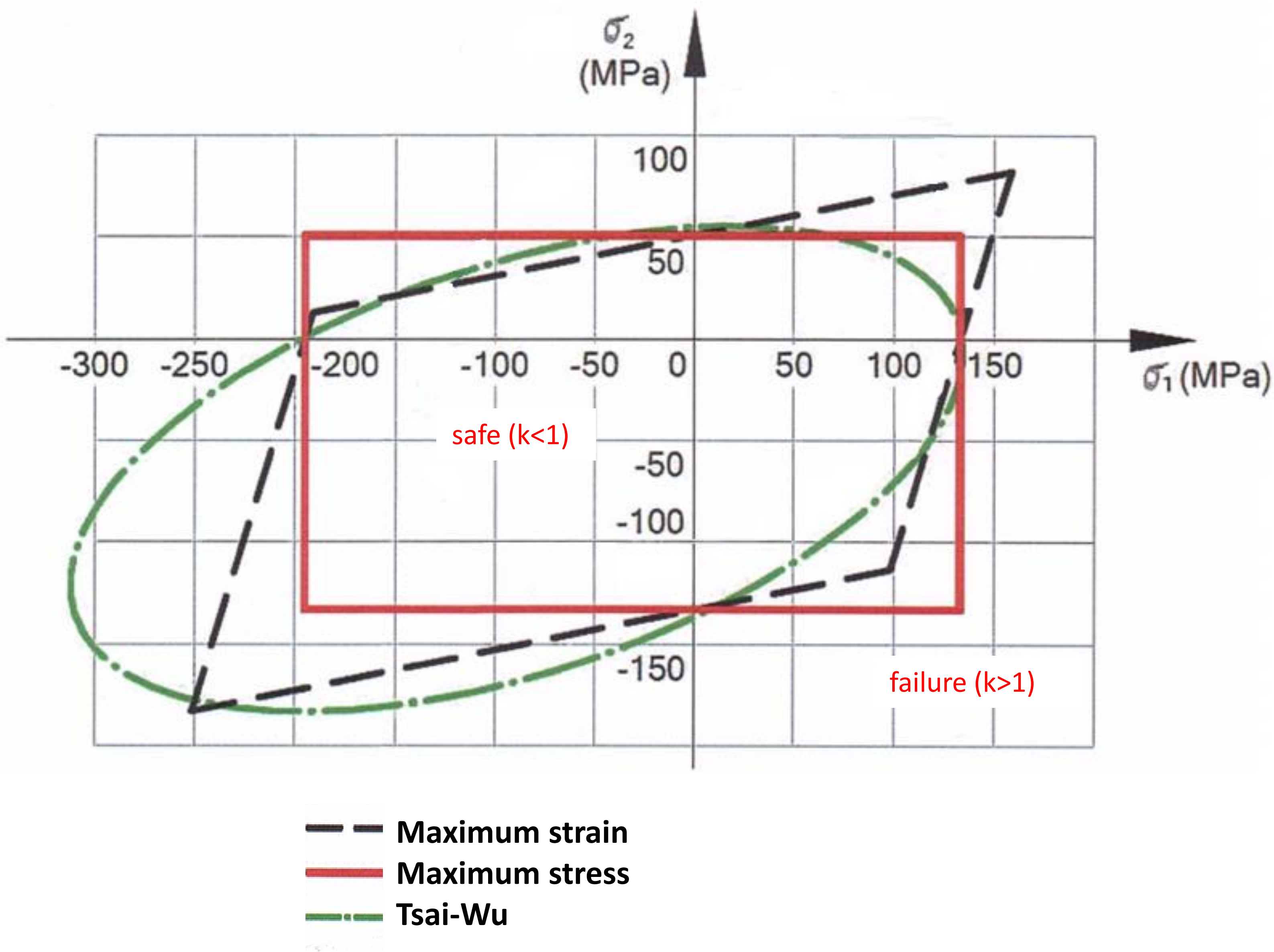
$$F_1 = \frac{1}{X_{t1}} + \frac{1}{X_{c1}}, \qquad F_{11} = \frac{-1}{X_{c1}X_{t1}}$$

$$F_2 = \frac{1}{X_{t2}} + \frac{1}{X_{c2}}, \qquad F_{22} = \frac{-1}{X_{t2}X_{c2}}$$

$$F_{66} = \frac{1}{S_{12}^w}, \quad F_6 = 0$$

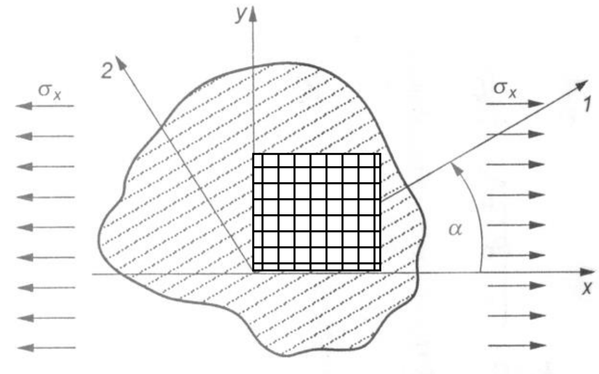
$$F_{12} = -\frac{\sqrt{F_{11} \cdot F_{22}}}{2}$$

Comparison of Stress, Strain, and Tsai–Wu criteria



Example 1. Unidirectional stretching of a single orthotropic layer

Material properties: $E_{11} = 2 \cdot 10^5\ MPa$, $E_{22} = 0{,}5 \cdot 10^5\ MPa$, $\nu_{12} = 0{,}25$, $G_{12} = 2 \cdot 10^4\ MPa$. The layer is loaded by $\sigma_{xx} = 10$ MPa. Find stress in xy-coordinate system as a function of α angle.



<u>Analytical solution:</u>

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} S_{11}^* & S_{12}^* & S_{16}^* \\ S_{12}^* & S_{22}^* & S_{26}^* \\ S_{16}^* & S_{26}^* & S_{66}^* \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ 0 \\ 0 \end{Bmatrix},$$

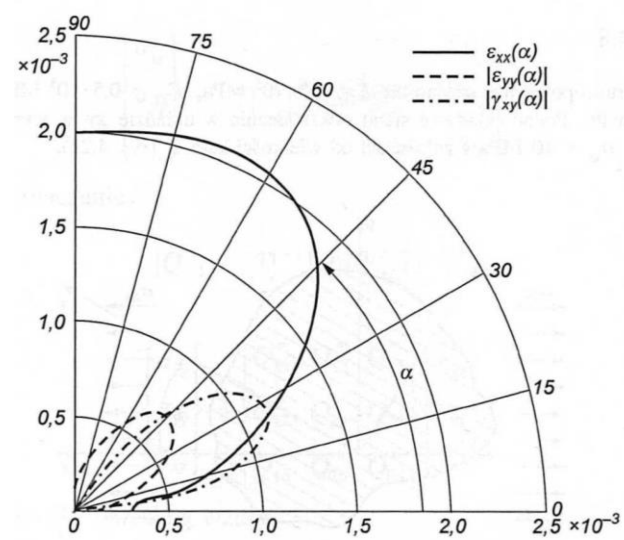
yields:

$$\varepsilon_{xx} = S_{11}^* \sigma_{xx},$$
$$\varepsilon_{yy} = S_{12}^* \sigma_{xx},$$
$$\gamma_{xy} = S_{16}^* \sigma_{xx},$$

where:

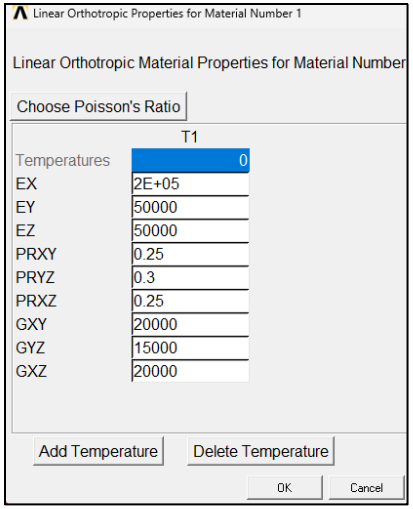
$$S_{11}^* = S_{11}c^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}s^4,$$
$$S_{12}^* = S_{12}c^4 + (S_{11} + S_{22} - S_{66})s^2c^2 + S_{12}s^4$$
$$S_{16}^* = (2S_{12} + S_{66} - 2S_{22})sc^3 - (2S_{12} + S_{66} - 2S_{11})s^3c.$$

$$S_{11} = \frac{1}{E_{11}}, \qquad S_{22} = \frac{1}{E_{22}}, \qquad S_{12} = \frac{-\nu_{21}}{E_{22}} = \frac{-\nu_{12}}{E_{11}}, \qquad S_{66} = G_{12}.$$



Strain components for a set of α values

α	0	5	15	30	45	60	75	85	90
$10^4\varepsilon_{xx}$	0,5	0,528	0,741	1,297	1,812	2,047	2,040	2,006	2,000
$10^4\varepsilon_{yy}$	-0,125	-0,142	-0,266	-0,547	-0,688	-0,547	-0,266	-0,142	-0,125
$10^4\gamma_{xy}$	0	-0,323	-0,862	-1.137	-0,75	-0,162	-0,112	-0,062	0

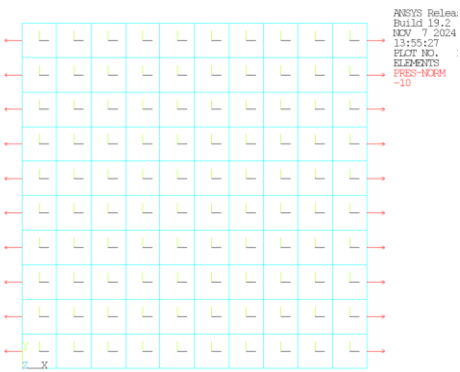


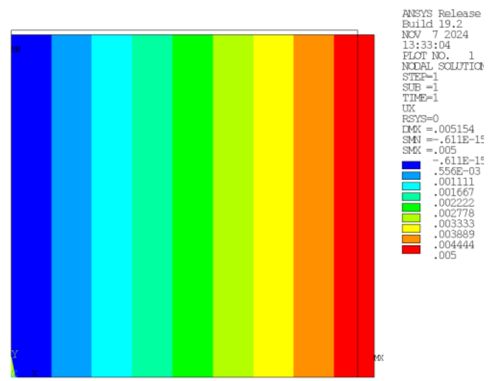
DOF Solution (ANSYS)

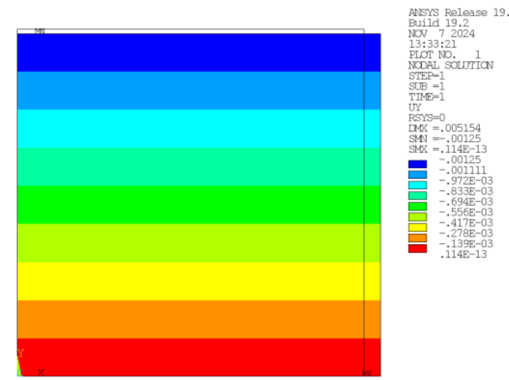
UX [mm]

UY [mm]

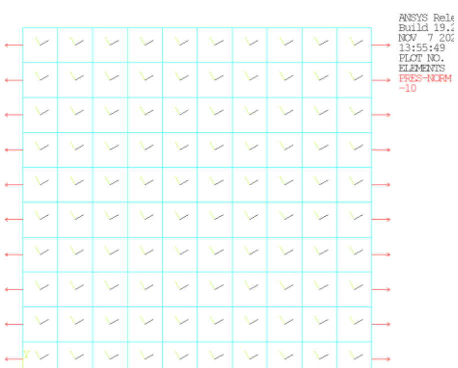
α = 0°

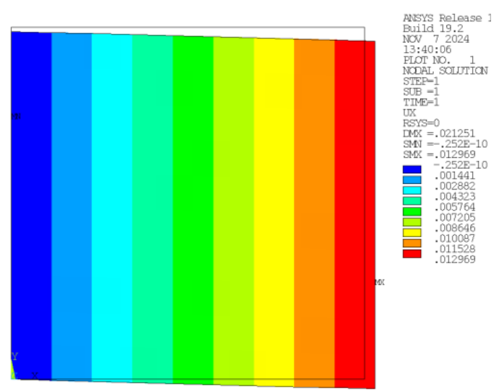


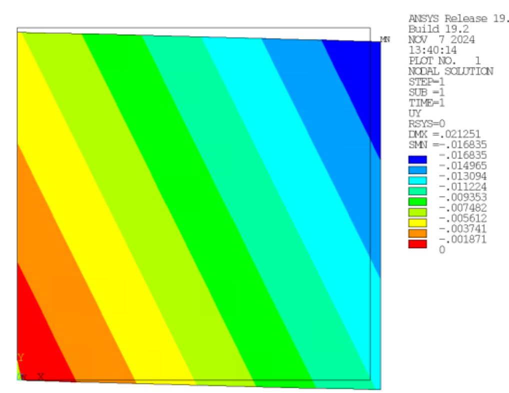




α = 30°



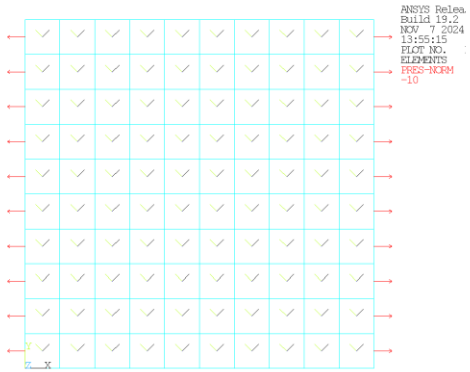
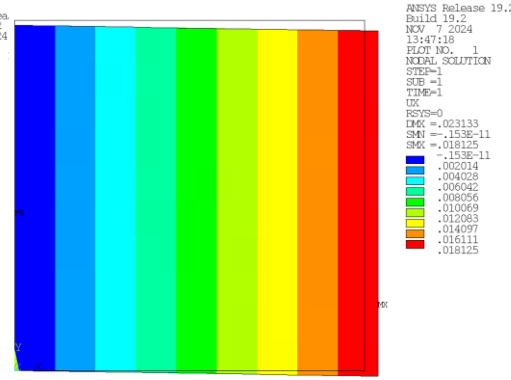
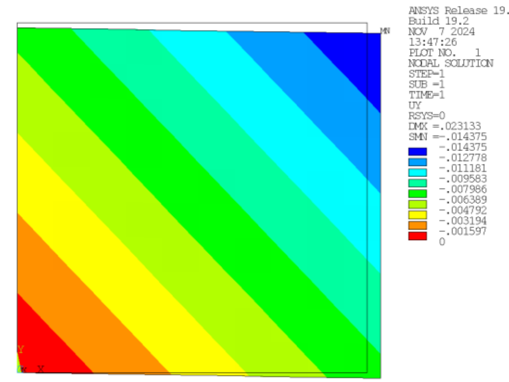




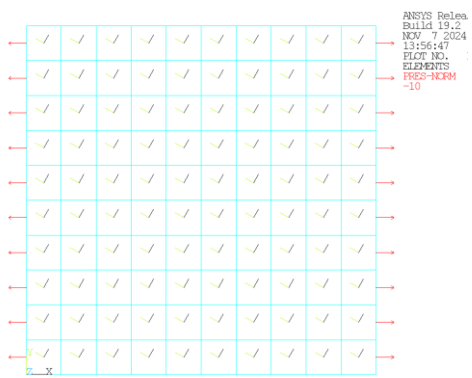
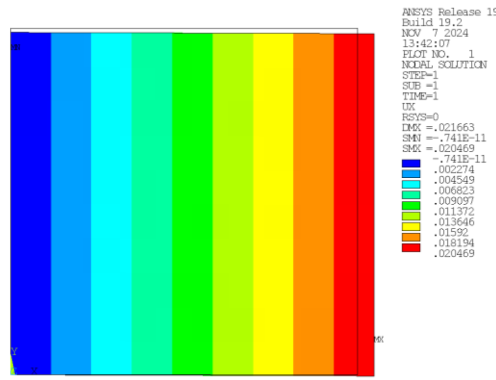
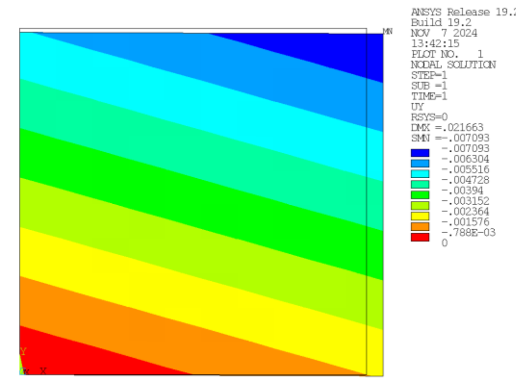
DOF Solution in ANSYS

UX [mm] | UY [mm]

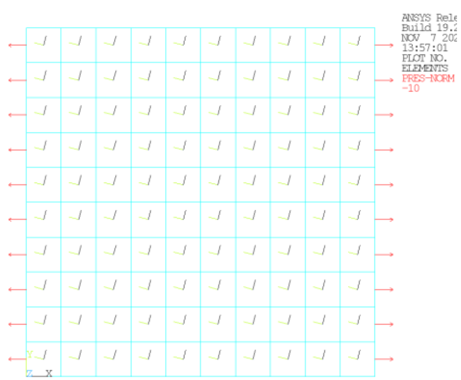
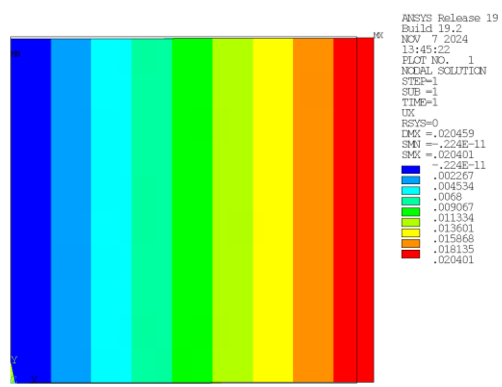
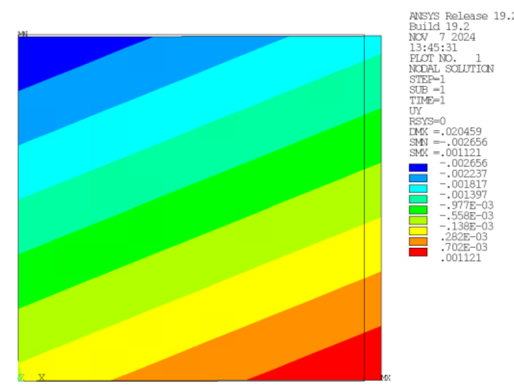
α = 45°

α = 60°

α = 75°

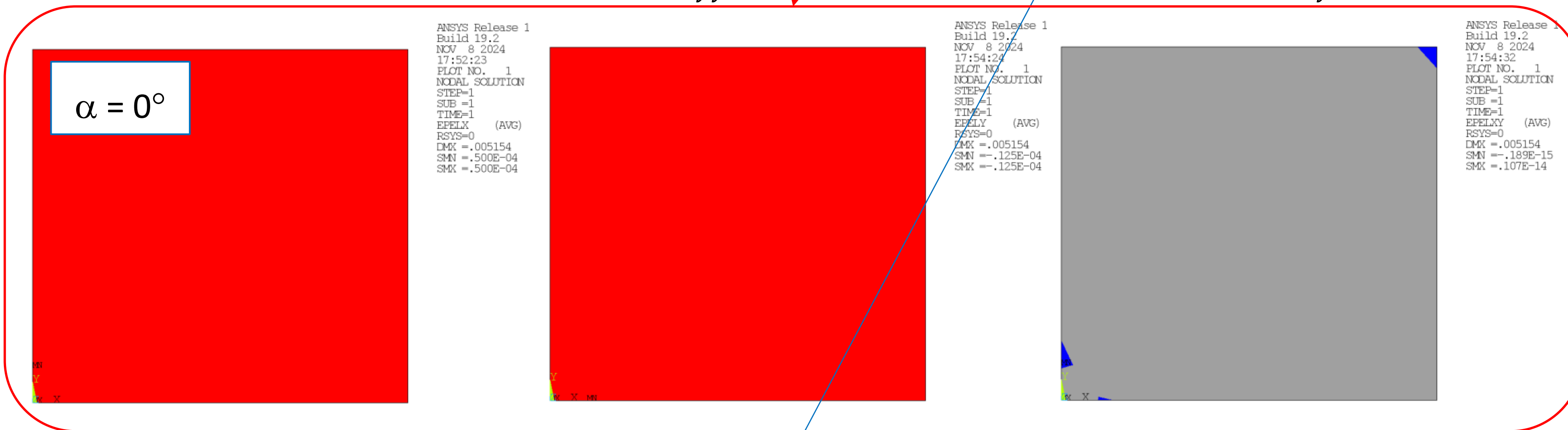




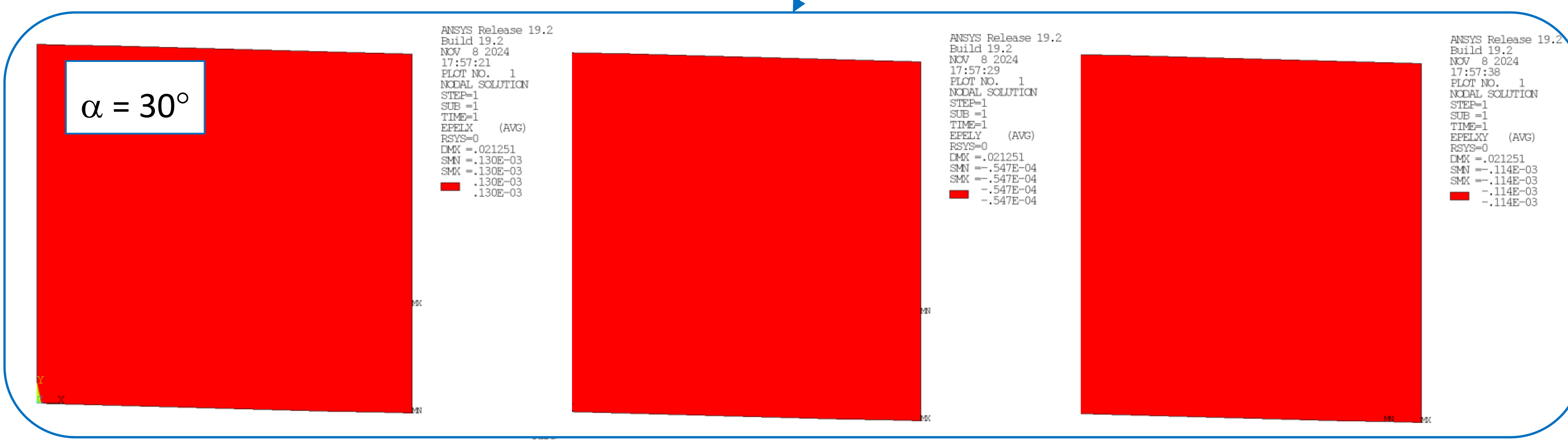
Strain

Nodal solution in ANSYS

α	0	5	15	30	45	60	75	85	90
$10^4\varepsilon_{xx}$	0,5	0,528	0,741	1,297	1,812	2,047	2,040	2,006	2,000
$10^4\varepsilon_{yy}$	-0,125	-0,142	-0,266	-0,547	-0,688	-0,547	-0,266	-0,142	-0,125
$10^4\gamma_{xy}$	0	-0,323	-0,862	-1.137	-0,75	-0,162	-0,112	-0,062	0

EPELX (ε_{xx}) EPELY (ε_{yy}) EPELXY (γ_{xy})





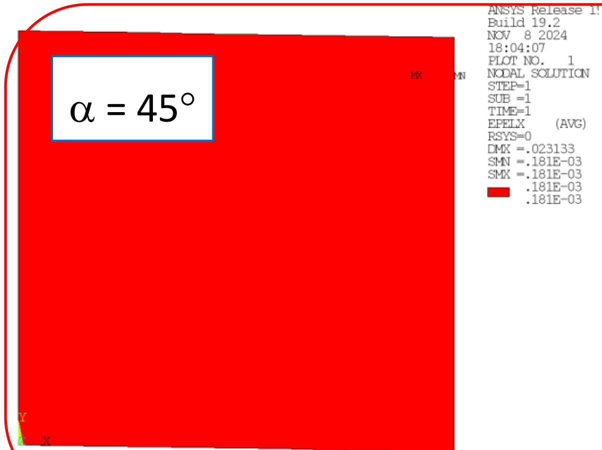
Strain

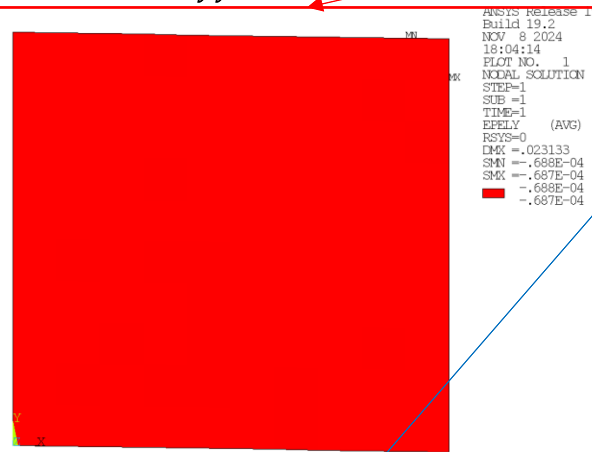
Nodal solution in ANSYS

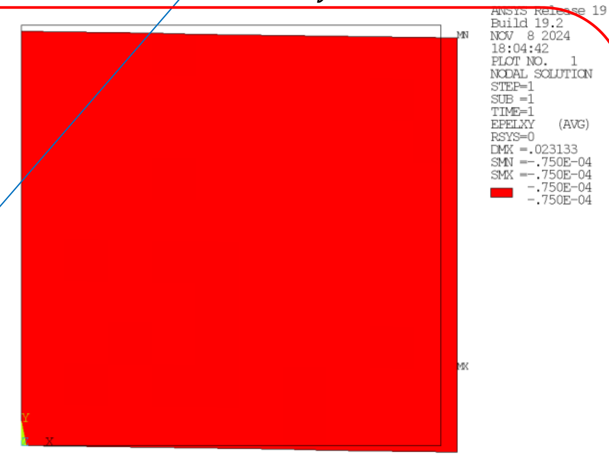
α	0	5	15	30	45	60	75	85	90
$10^4\varepsilon_{xx}$	0,5	0,528	0,741	1,297	1,812	2,047	2,040	2,006	2,000
$10^4\varepsilon_{yy}$	-0,125	-0,142	-0,266	-0,547	-0,688	-0,547	-0,266	-0,142	-0,125
$10^4\gamma_{xy}$	0	-0,323	-0,862	-1.137	-0,75	-0,162	-0,112	-0,062	0

EPELX (ε_{xx}) EPELY (ε_{yy}) EPELXY (γ_{xy})

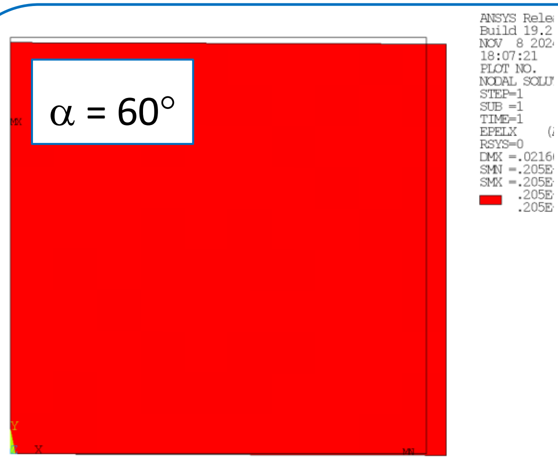
α = 45°

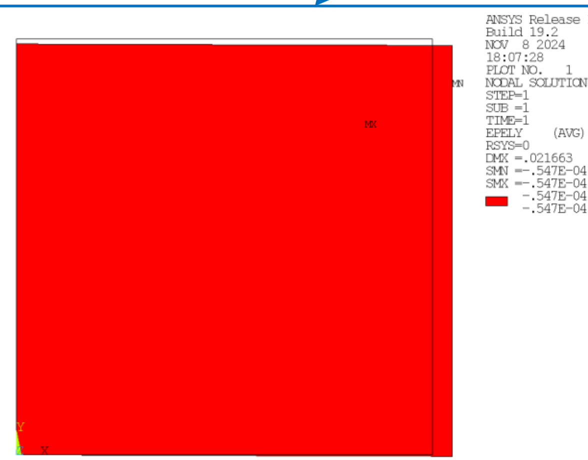


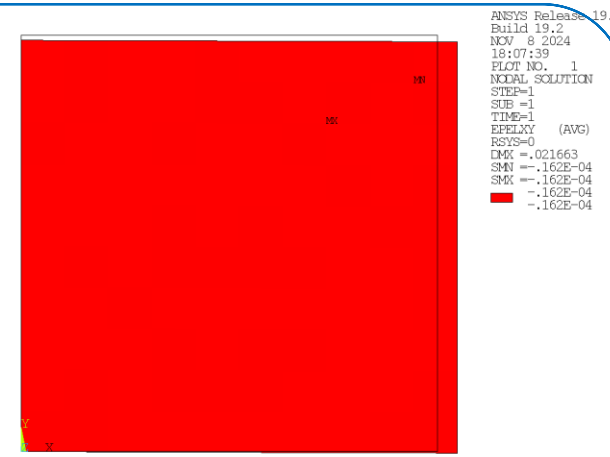




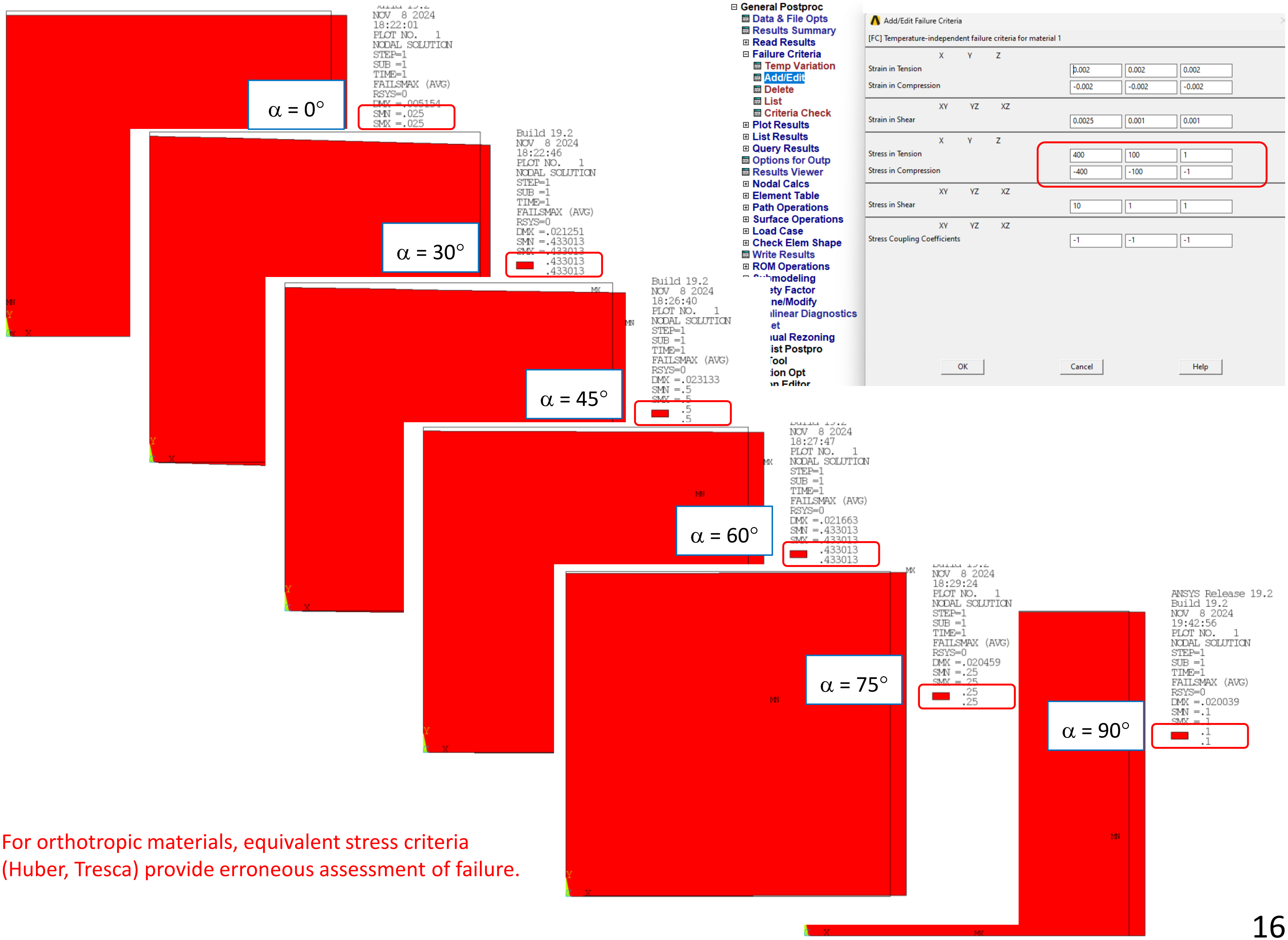
α = 60°





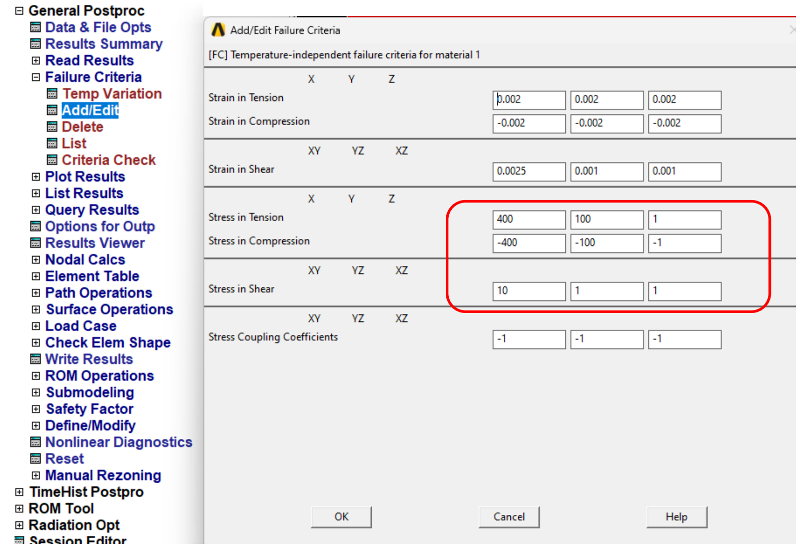


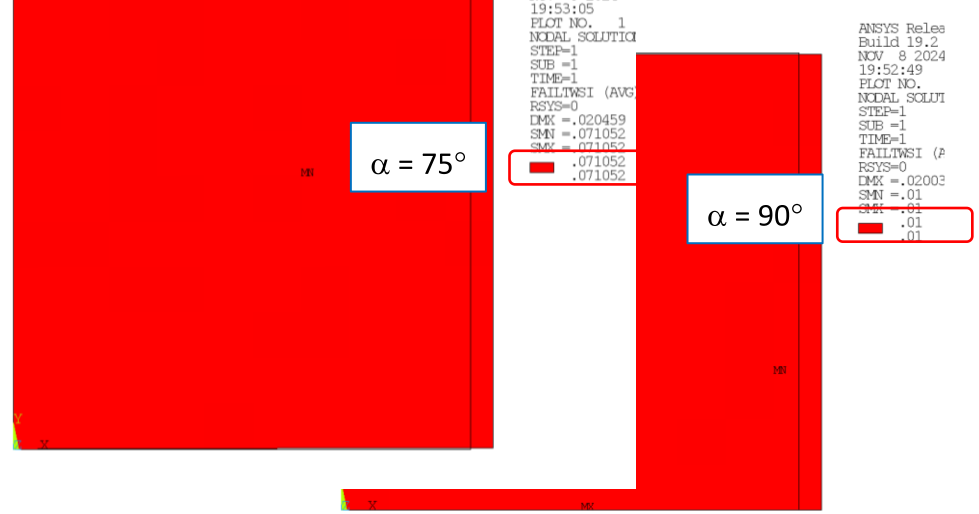
Maximum stress criterion



For orthotropic materials, equivalent stress criteria (Huber, Tresca) provide erroneous assessment of failure.

Tsai Wu criterion



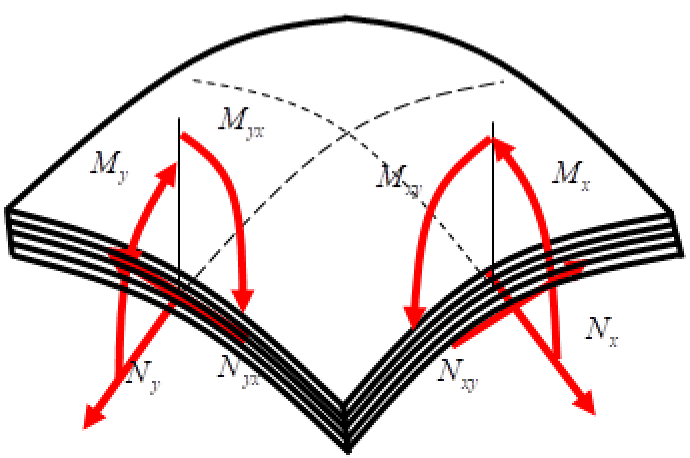


For orthotropic materials, equivalent stress criteria (Huber, Tresca) provide erroneous assessment of failure.

BASICS OF MULTILAYER COMPOSITES

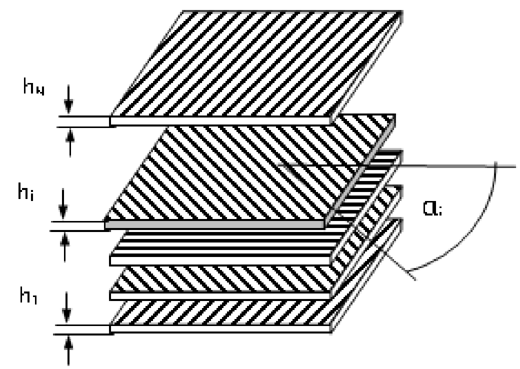
Assumptions:

- the laminate is composed of multiple orthotropic layers tightly bonded together,
- stresses perpendicular to the layers can be ignored for thin laminates (thickness << length and width),
- strain components are continuous at the interface between layers,
- stresses discontinuities are allowable due to differences in material properties,
- small deformations,
- the line normal to the laminate surface remains normal to it after deformation (Kirchoff's hypothesis)



Internal forces in a multilayer composite

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix}$$



where:

$\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0$ – strain components in the middle layer,

$\kappa_x = \frac{\partial^2 w_0}{\partial x^2}, \kappa_y = \frac{\partial^2 w_0}{\partial y^2}, \kappa_{xy} = \frac{\partial^2 w_0}{\partial x \partial y}$ - curvatures of the middle layer,

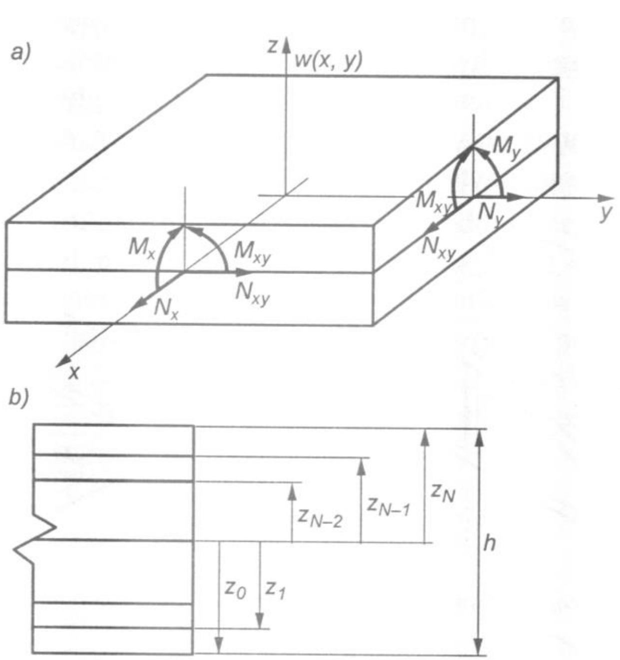
w_0 – deflection of the middle layer,

$N_x, N_y, N_{xy}, M_x, M_y, M_{xy}$ – internal forces,

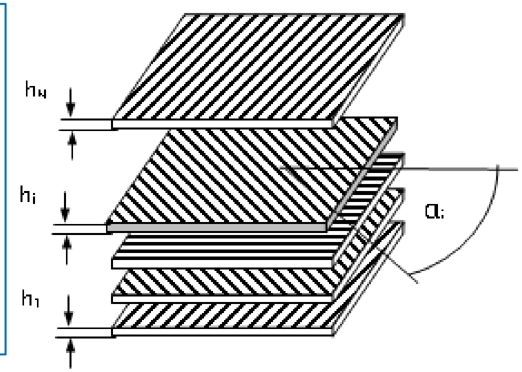
A – in-plane laminate stiffness matrix, ***B*** – coupling stiffness matrix,

D – flexural-torsional stiffness matrix.

BASICS OF MULTILAYER COMPOSITES



$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix}$$



Matrices ***A***, ***B***, ***D*** (3×3), are symmetric and depend on material properties of layers, their orientation and location.

$$\boldsymbol{A}_{ij} = \sum_{k=1}^{N} Q_{ij}^{*k} \left(z_k - z_{k-1}\right)$$

$$\boldsymbol{B}_{ij} = \frac{1}{2}\sum_{k=1}^{N} Q_{ij}^{*k} \left(z_k^2 - z_{k-1}^2\right)$$

$$\boldsymbol{D}_{ij} = \frac{1}{3}\sum_{k=1}^{N} Q_{ij}^{*k} \left(z_k^3 - z_{k-1}^3\right)$$

where: z_k – distance of k-layer from the middle layer
Q_{ij}^{*k} – stiffnes matrix of k-layer transformed to xy-coordinate system

Example 2. Unidirectional stretching of a six-layer laminate

Thickness of each layer is 0.125 mm, reinforcement directions are [45°,-45°,45°,-45°,45°,-45°]. Size: L_x = 100 mm, L_y = 100 mm. A carbon-epoxy composite has Young's moduli: E_{11}= 211000 MPa, E_{22}= E_{33}= 5300 MPa, Kirchoff's moduli G_{12}= G_{31}= 2600 MPa, G_{23}= 2000 MPa. Poisson's ratios: ν_{12}= ν_{31}= 0.25, ν_{23}= 0.2 . Load is applied as uniform traction $\mathbf{N_x}$ **=** 10N/mm. Find deformation and stresses in layers.

Analytical solution:

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} 1.348e-4 & -1.216e-4 & 0 & 0 & 0 & -1.69e-5 \\ -1.216e-4 & 1.348e-4 & 0 & 0 & 0 & -1.69e-5 \\ 0 & 0 & 2.685e-5 & -1.69e-5 & -1.69e-5 & 0 \\ 0 & 0 & -1.69e-5 & 2.876e-3 & -2.594e-3 & 0 \\ 0 & 0 & -1.69e-5 & -2.594e-3 & 2.876e-3 & 0 \\ -1.69e-5 & -1.69e-5 & 0 & 0 & 0 & 5.728e-4 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Strain in the middle layer:

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} 1.348e-3 \\ -1.216e-3 \\ 0 \\ 0 \\ 0 \\ -1.69e-4 \end{bmatrix}$$

Displacements (assuming small deformations):

$Ux = \varepsilon_x^0\, Lx = 0{,}135$ mm,

$Uy = \varepsilon_y^0\, L_y = -0{,}122$ mm,

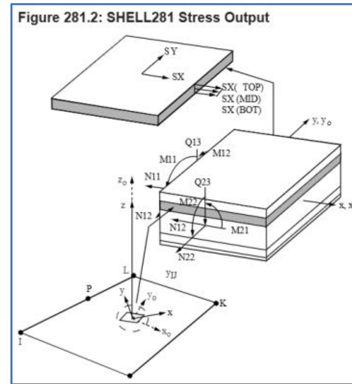
$Uz = \kappa_{xy} \frac{L_x L_y}{2} = -0{,}845$ mm.

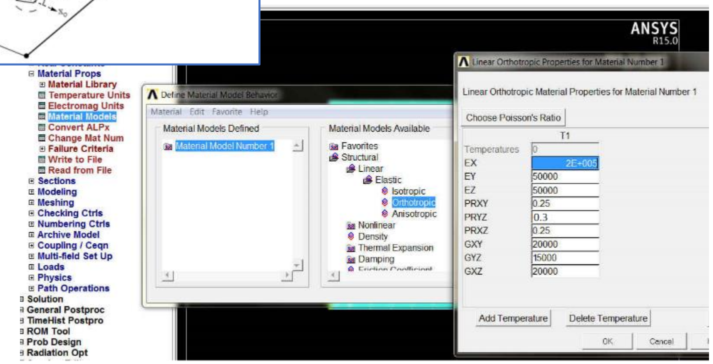
FE model of a six-layer laminate

Analytical solution:

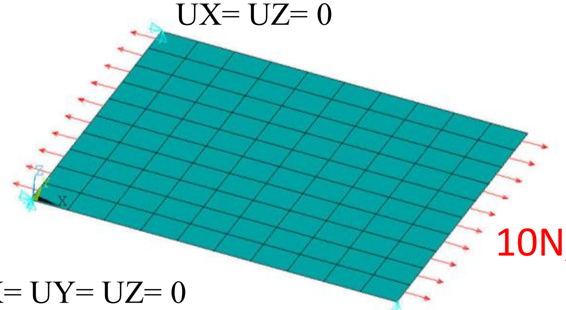
$Ux = \varepsilon_x^0 \, Lx = 0{,}135$ mm,
$Uy = \varepsilon_y^0 \, L_y = -0{,}122$ mm,
$Uz = \kappa_{xy} \frac{L_x L_y}{2} = -0{,}845$ mm.

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} 1.348e-3 \\ -1.216e-3 \\ 0 \\ 0 \\ 0 \\ -1.69e-4 \end{bmatrix}$$





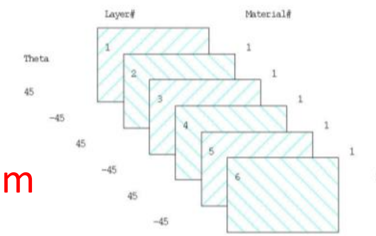
UX= UZ= 0

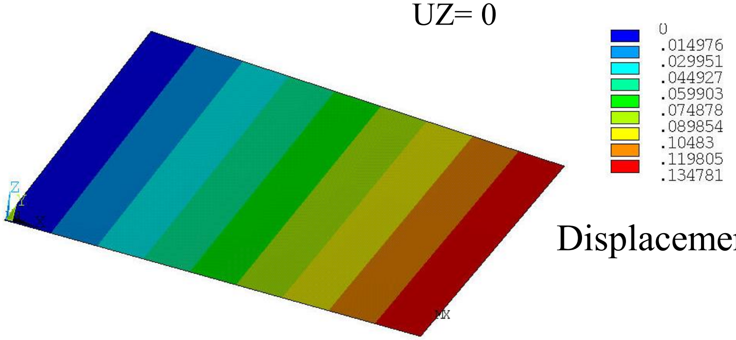


10N/mm

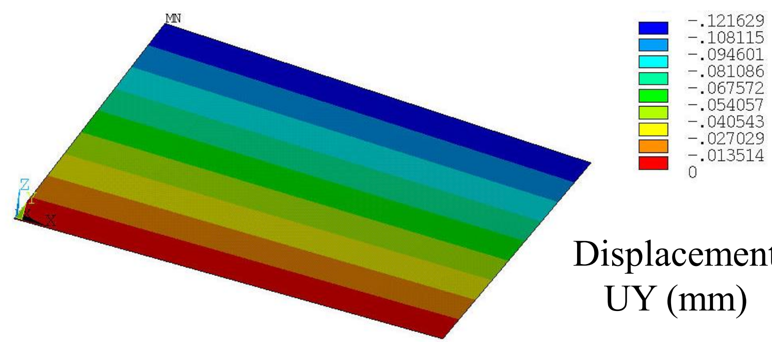
UX= UY= UZ= 0

UZ= 0

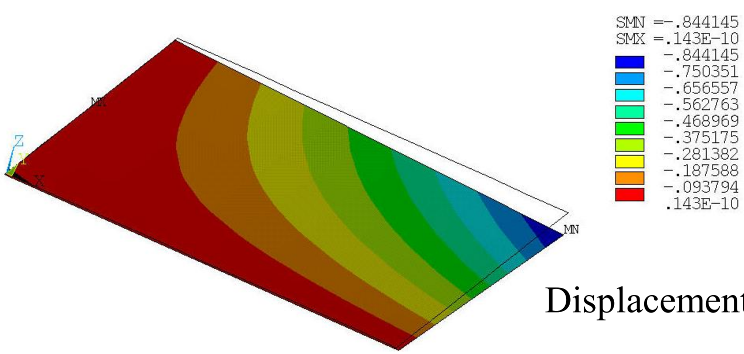




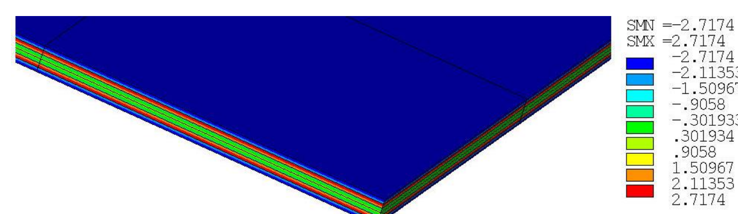
Displacement UX (mm)



Displacement UY (mm)



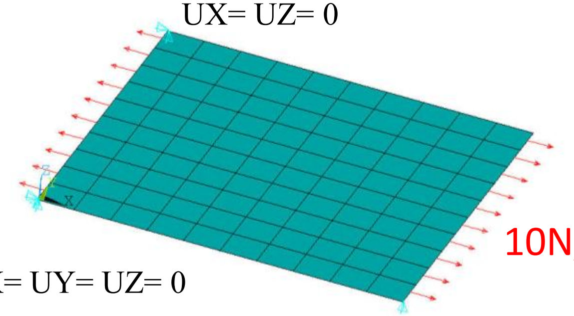
Displacement UZ (mm)



Stress SY in layers (σ_y)

Example 3. Unidirectional stretching of a four-layer laminate

UX= UZ= 0



10N/mm

UX= UY= UZ= 0

UZ= 0

The limit values of stress and strain

Add/Edit Failure Criteria

[FC] Temperature-independent failure criteria for material 1

	X	Y	Z
Strain in Tension	0.0019	0.0019	0.001
Strain in Compression	-0.0019	-0.0019	-0.001

	XY	YZ	XZ
Strain in Shear	0.0025	0.001	0.001

	X	Y	Z
Stress in Tension	400	10	1
Stress in Compression	-400	-10	-1

	XY	YZ	XZ
Stress in Shear	10	1	1
Stress Coupling Coefficients	-1	-1	-1

Material properties

Linear Orthotropic Properties for Material Numb

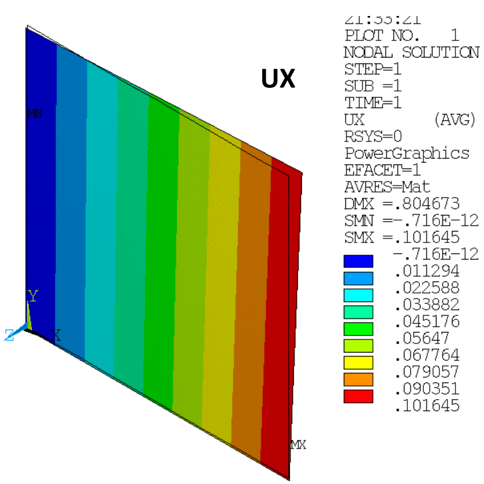
Linear Orthotropic Material Properties f

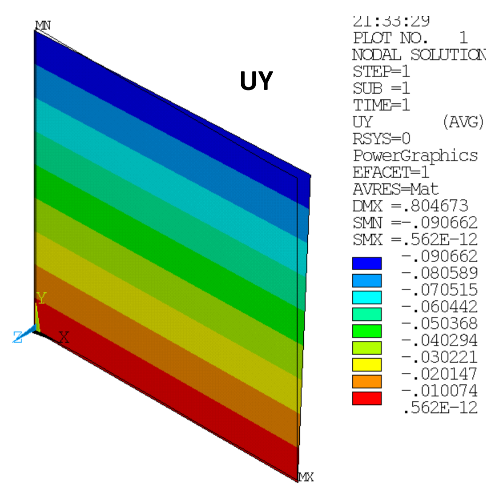
Choose Poisson's Ratio

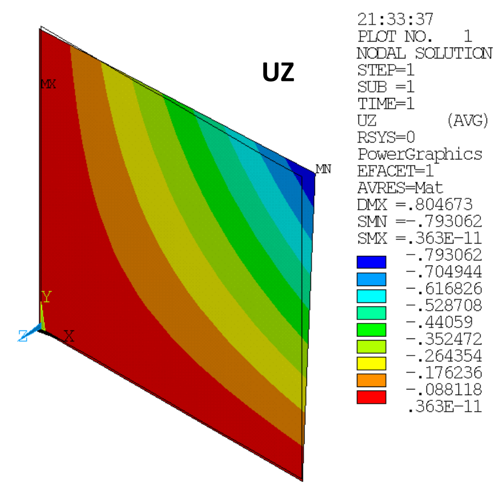
	T1
Temperatures	0
EX	2.11E+05
EY	5300
EZ	1000
PRXY	0.25
PRYZ	0
PRXZ	0
GXY	2600
GYZ	1000
GXZ	1000

Add Temperature   Delete Temp

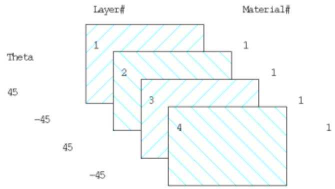
DOF Solution



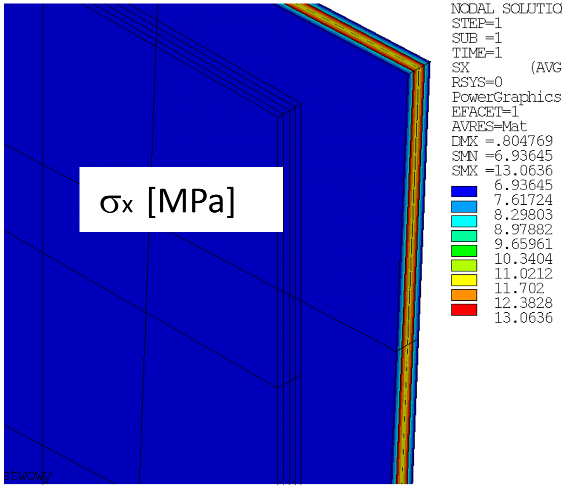


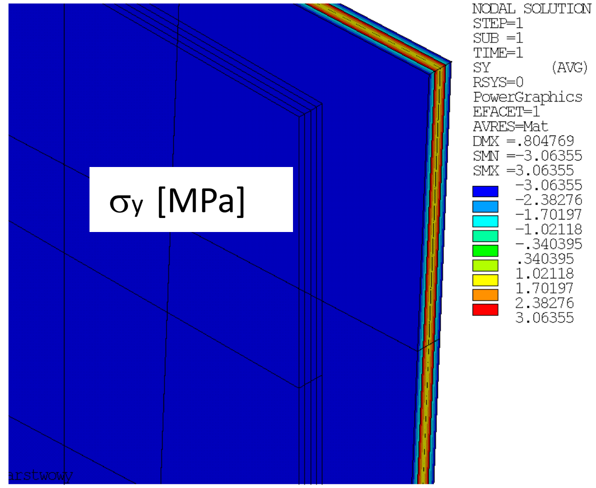


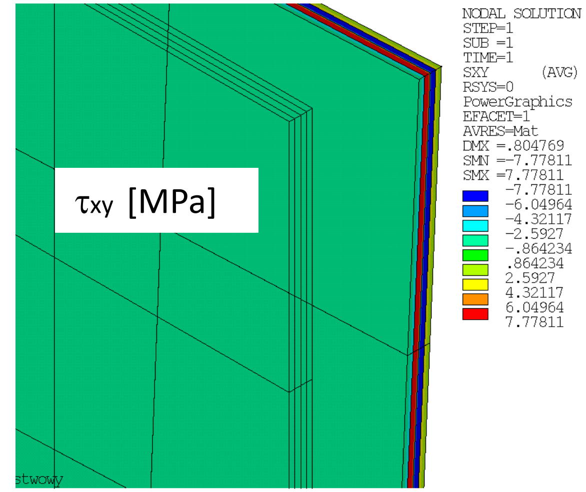
Stress components in layers *(two types of reinforcement)*

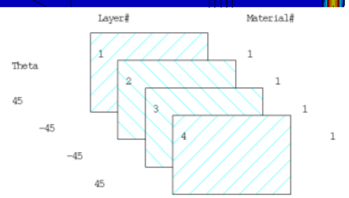


[45°,-45°,45°,-45°] – asymmetrical

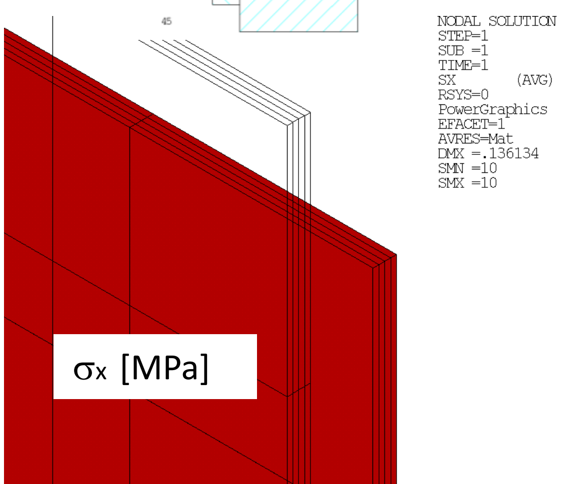


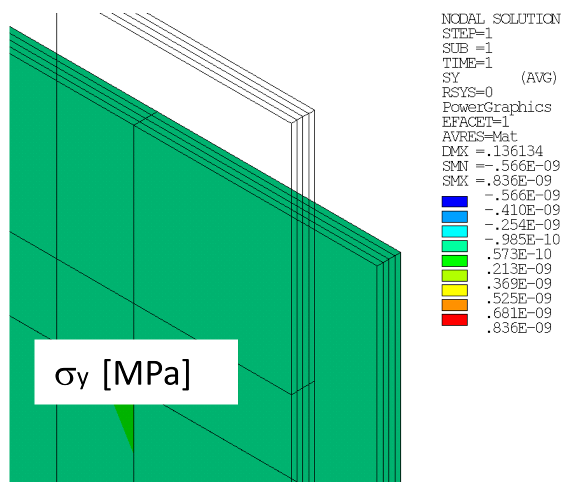


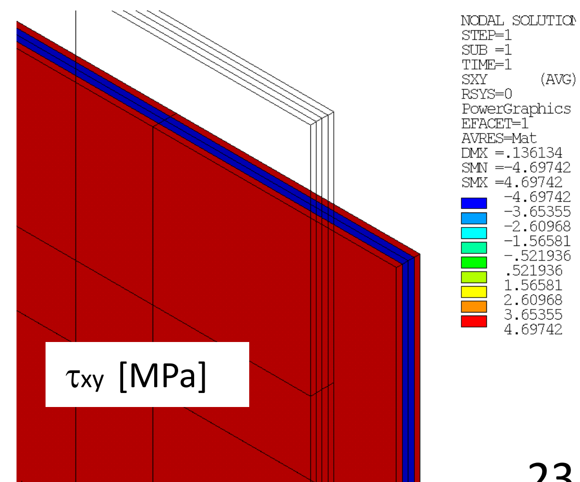




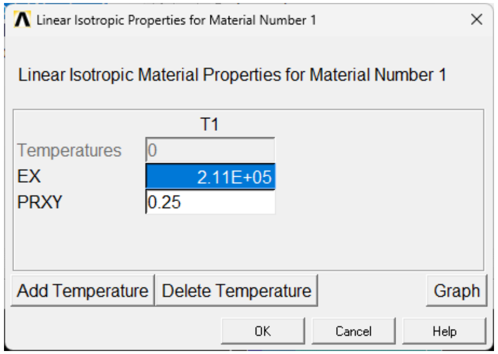
[45°,-45°,-45°,45°] – symmetrical



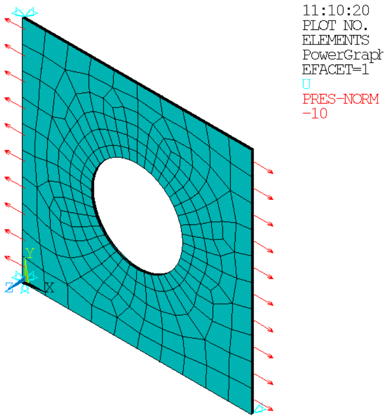




Example 4a. An isotropic plate with a hole (ϕ = 40mm)

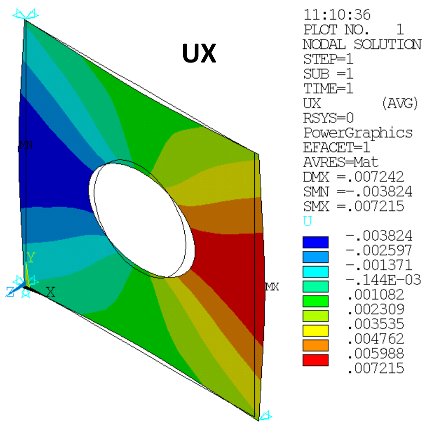


FE model

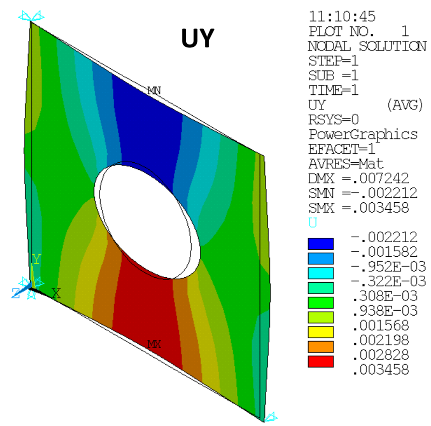


DOF Solution

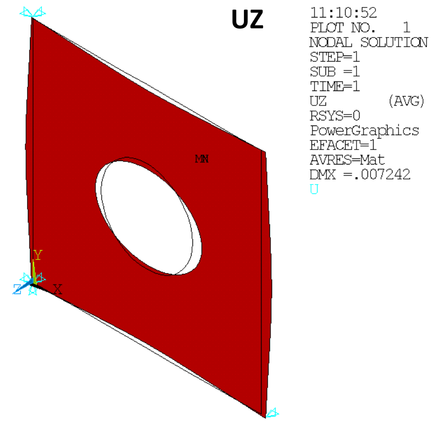
UX



UY

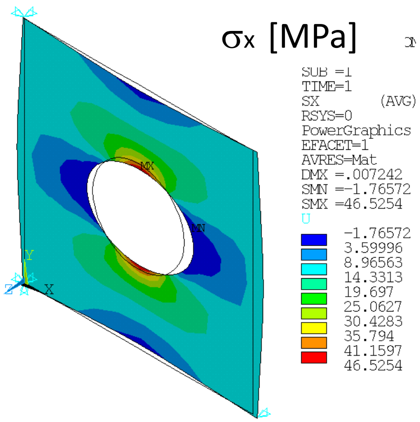


UZ

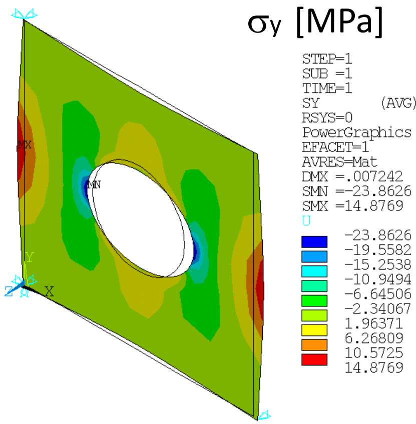


Stress

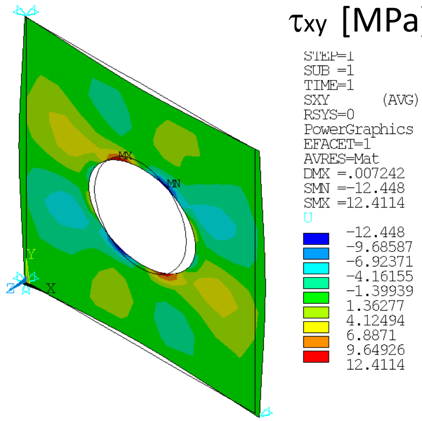
σ_x [MPa]



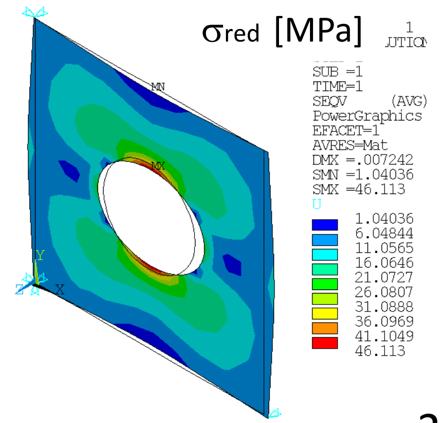
σ_y [MPa]



τ_{xy} [MPa]

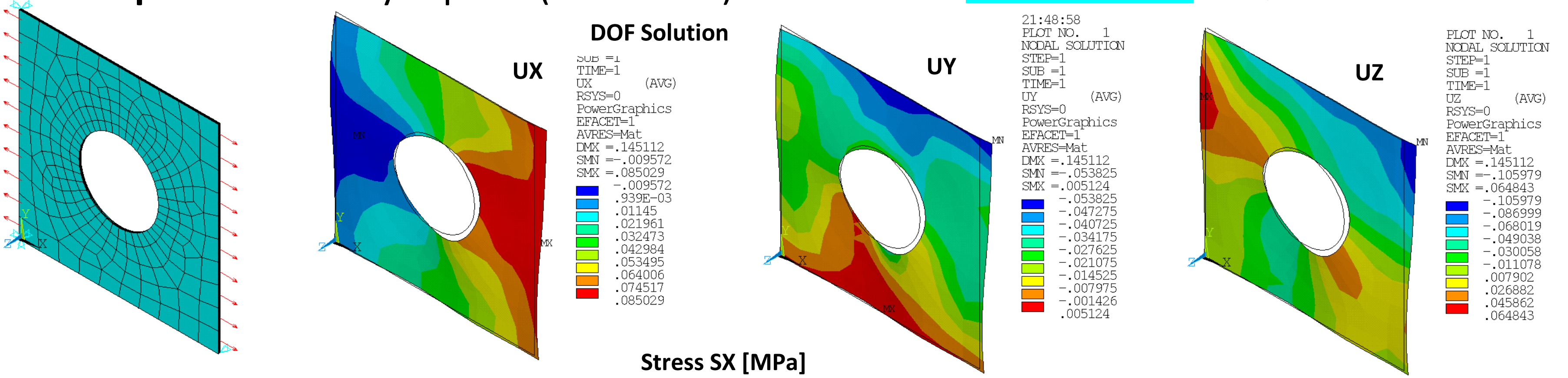


σ_{red} [MPa]

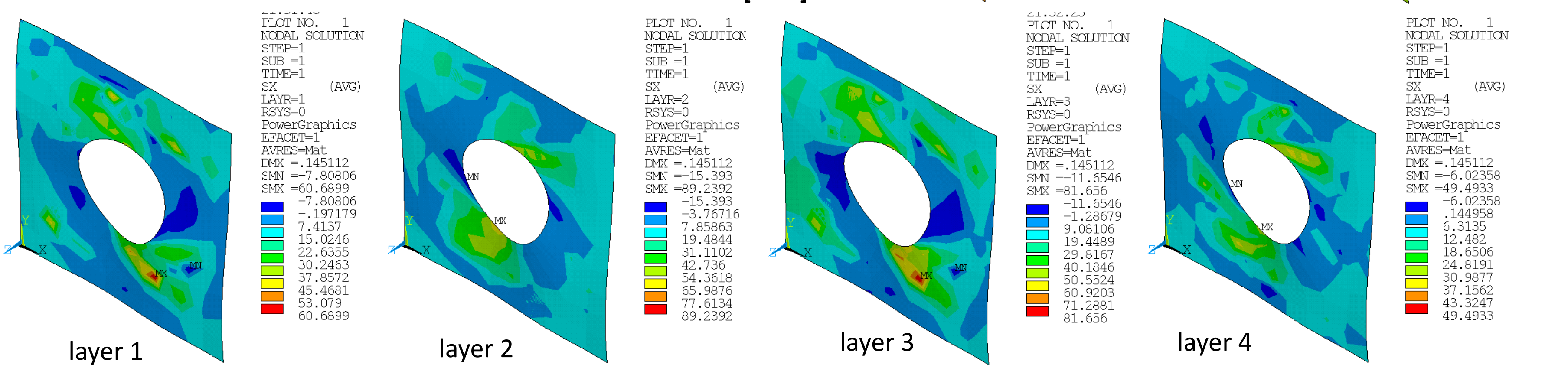


Example 4b. Four-layer plate (from Ex. 3) with a hole [45°,-45°,45°,-45°] – asymmetric reinforcement

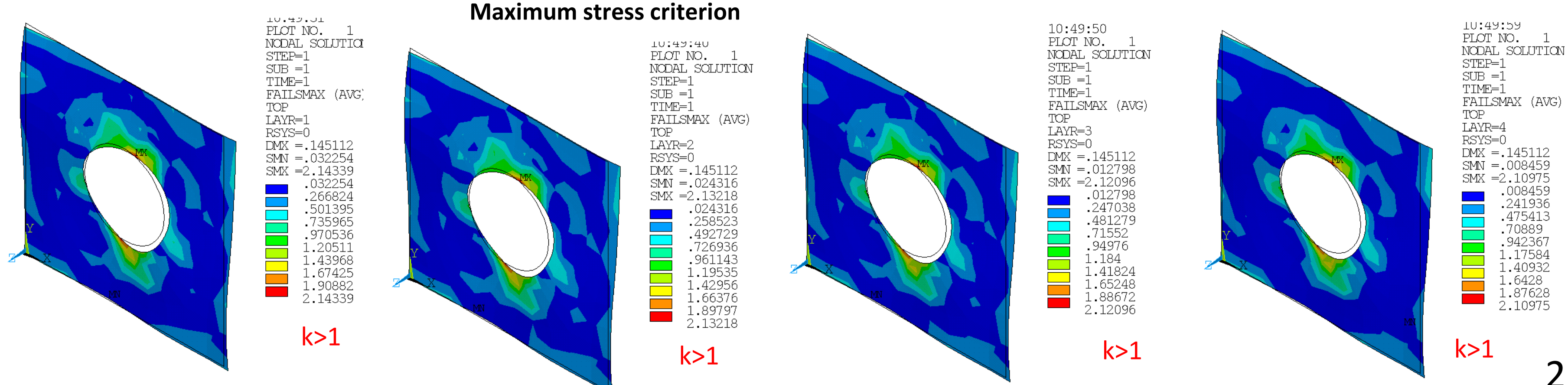
DOF Solution



Stress SX [MPa]

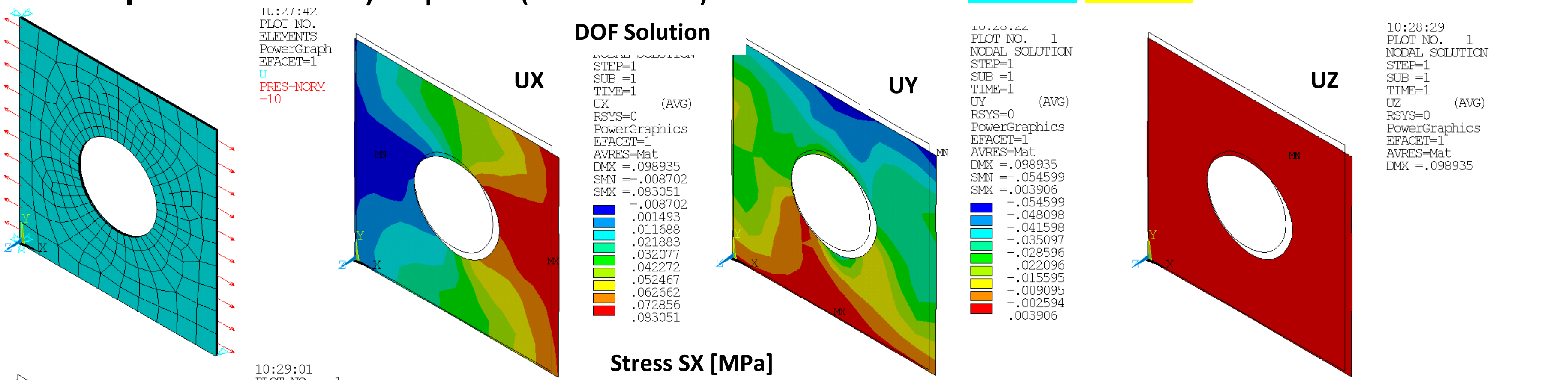


Maximum stress criterion

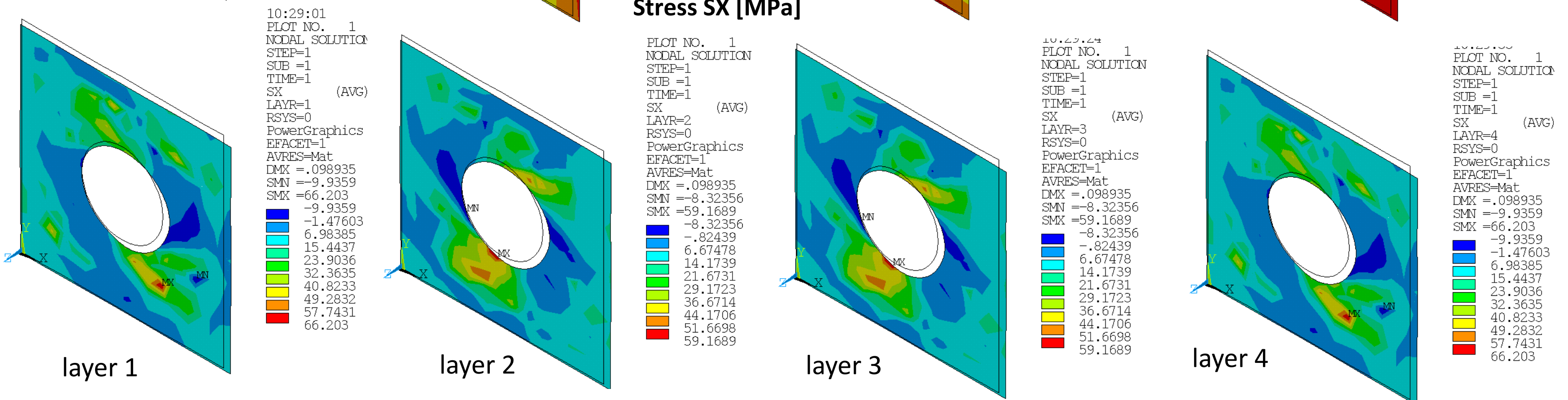


Example 4c. Four-layer plate (from Ex. 3) with a hole [45°,-45°,-45°,45°] – symmetric reinforcement

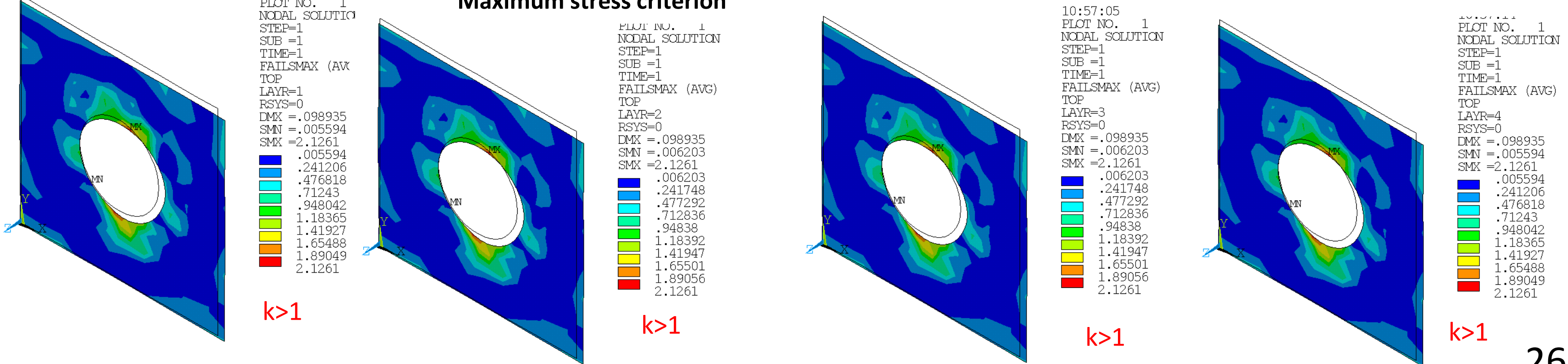
DOF Solution



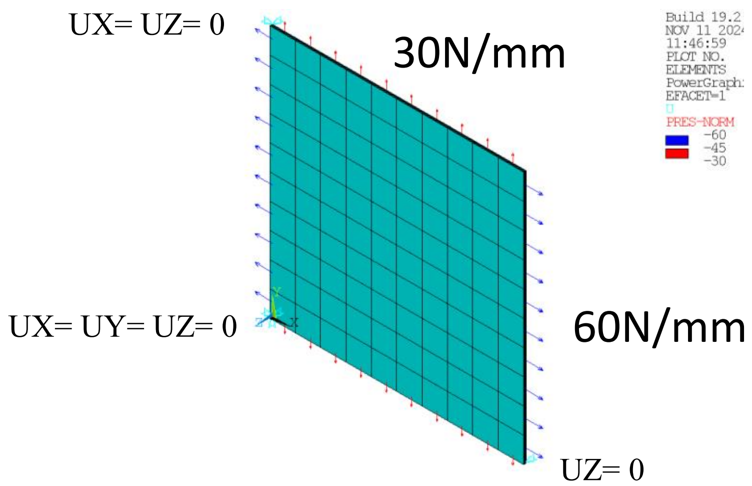
Stress SX [MPa]

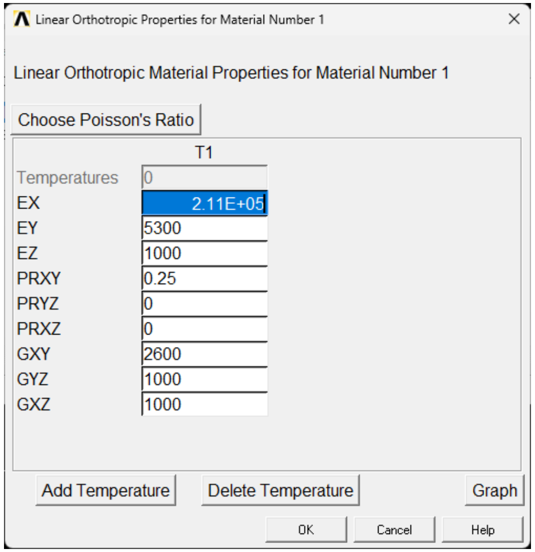


Maximum stress criterion

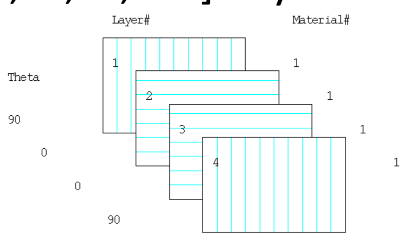


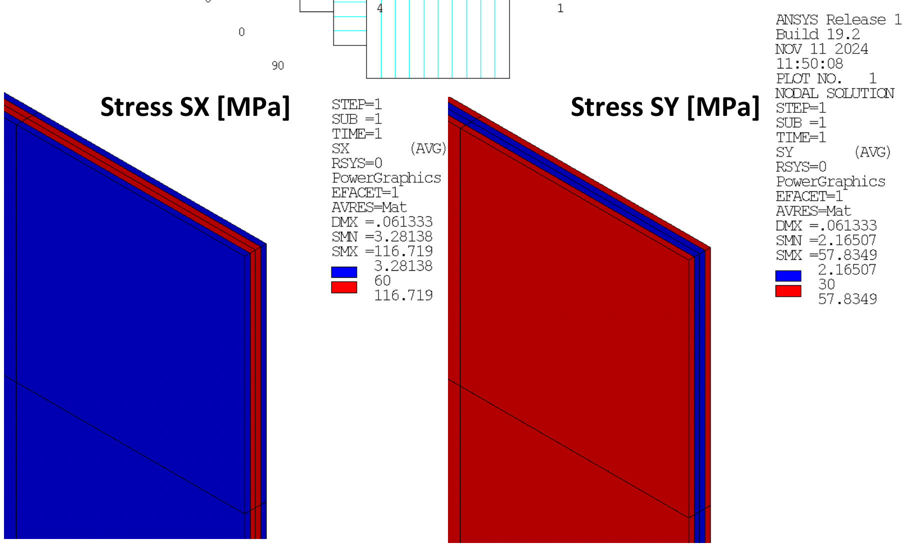
Example 5. 2D-stretching of a four-layer plate (from Ex. 3)



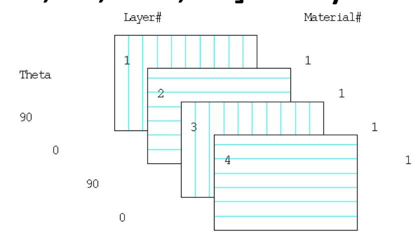


[90°,0°,0°,90°] - symmetric

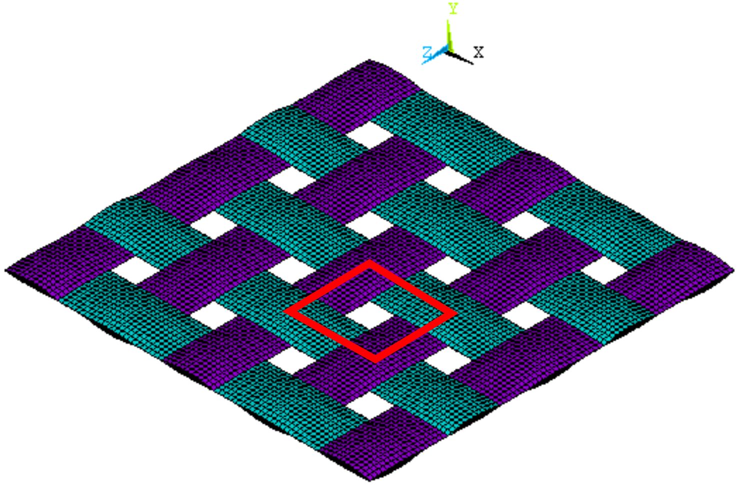


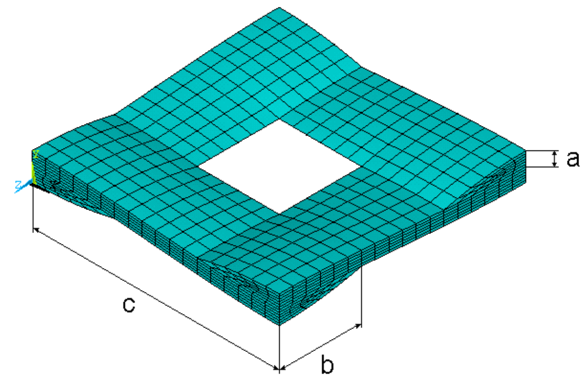


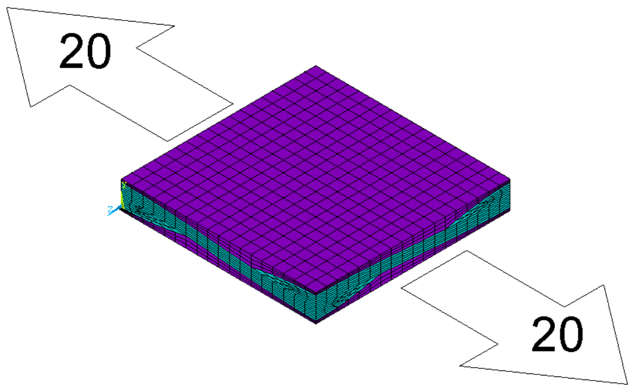
[90°,0°,90°,0°] - asymmetric

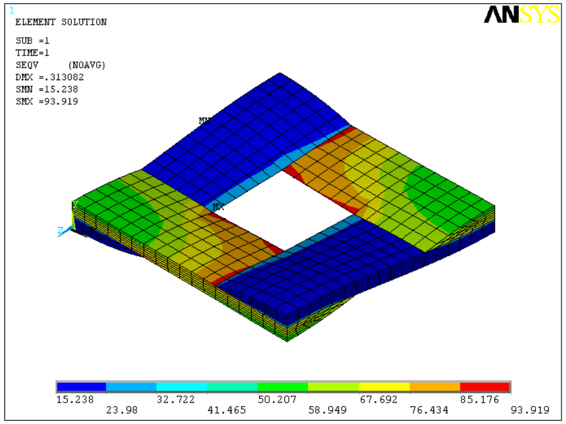


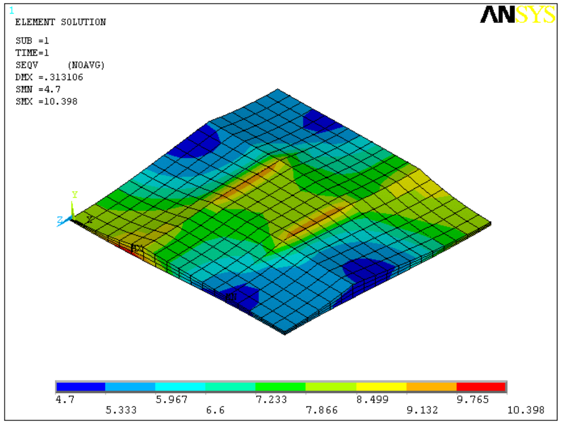
Example 6. Detailed modeling of a single layer (homogenization).











RVE- representative volume element

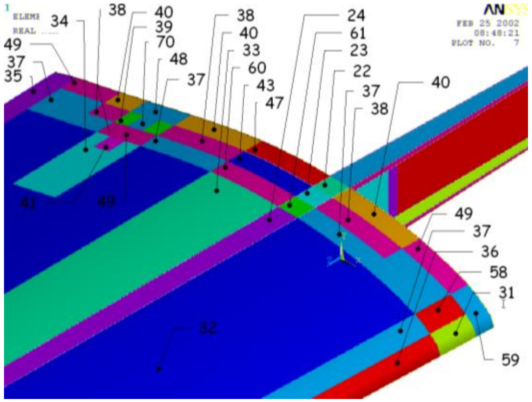
Von Mises stress in fibers [MPa]

Von Mises stress in matrix [MPa]

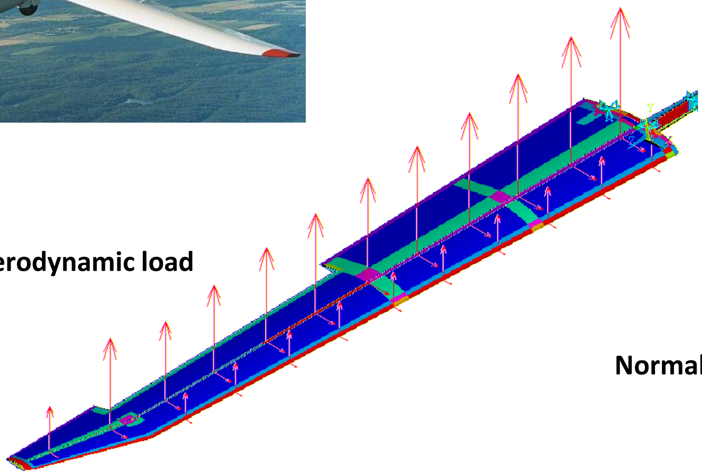
Homogenization replaces a complex heterogeneous model with a homogeneous material having substitute material properties.

Example 7. FEM analysis of the PW-5 glider wing

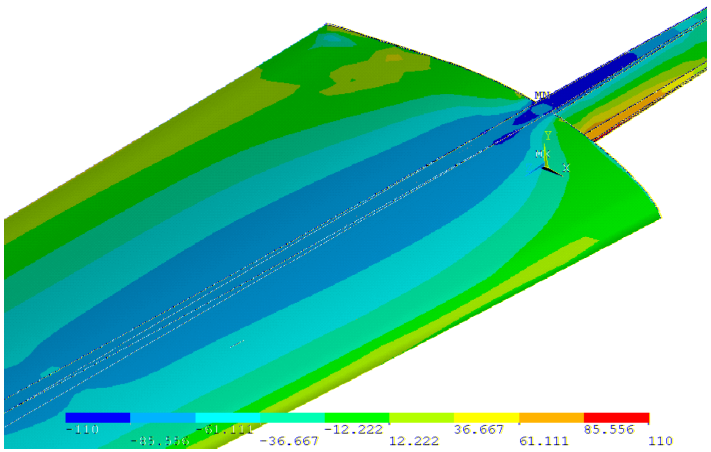
Reinforcement distribution



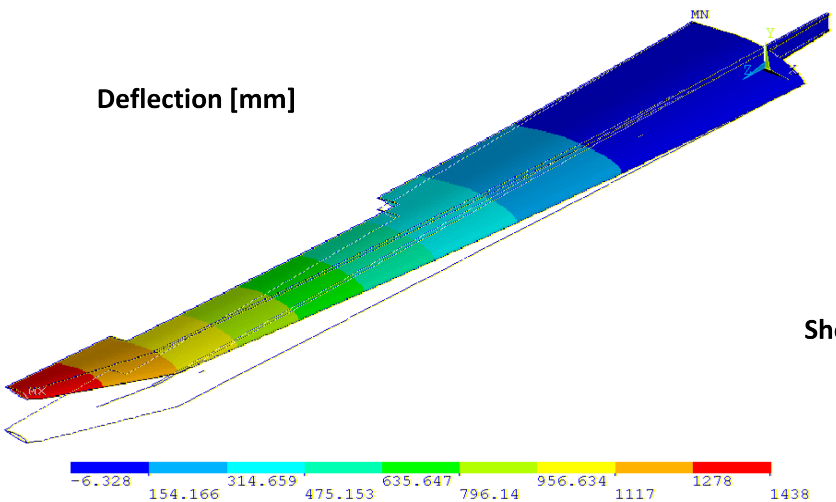
Aerodynamic load



Normal stress [MPa]



Deflection [mm]



Shear stress [MPa]