



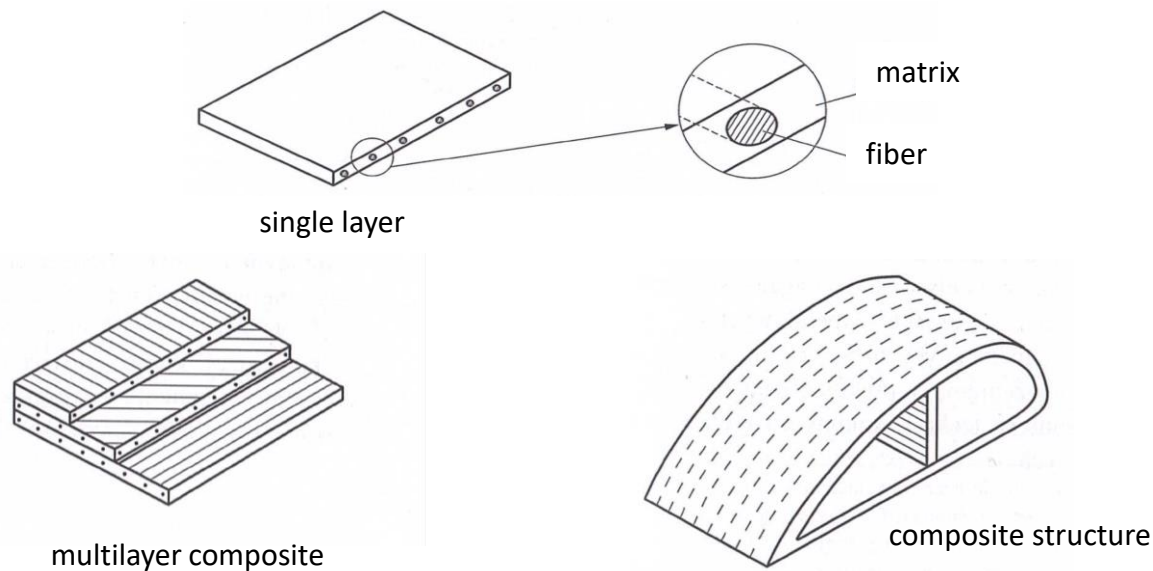
Institute of Aeronautics and Applied Mechanics

Finite element method 2 (FEM 2)

Composite structures

BASIC PROPERTIES OF COMPOSITE STRUCTURES

- heterogeneity
- direction-dependent material properties (anisotropy, orthotropy)
- complexity of constitutive laws
- multiscale modeling (from micro- to macroscale)
- main advantage: structures with properties optimized for given loads



Ortotropic material

Three perpendicular symmetry planes of material properties are a very common case in engineering practice. In this case, the number of independent coefficients in the constitutive matrix is reduced to 9, and the material is called orthotropic:

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_{11}} & -\frac{\nu_{21}}{E_{22}} & -\frac{\nu_{31}}{E_{33}} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_{11}} & \frac{1}{E_{22}} & -\frac{\nu_{32}}{E_{33}} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_{11}} & -\frac{\nu_{23}}{E_{22}} & \frac{1}{E_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}$$

E_{11}, E_{22}, E_{33} - Young's moduli in directions: 1, 2, 3

G_{12}, G_{23}, G_{31} - shear (Kirchoff's) moduli in planes: 12, 23 and 31

$\nu_{12}, \nu_{21}, \nu_{23}, \nu_{32}, \nu_{31}, \nu_{13}$ - Poisson's ratios

Due to symmetry:

$$\frac{\nu_{12}}{E_{11}} = \frac{\nu_{21}}{E_{22}}, \quad \frac{\nu_{13}}{E_{11}} = \frac{\nu_{31}}{E_{33}}, \quad \frac{\nu_{23}}{E_{22}} = \frac{\nu_{32}}{E_{33}}.$$

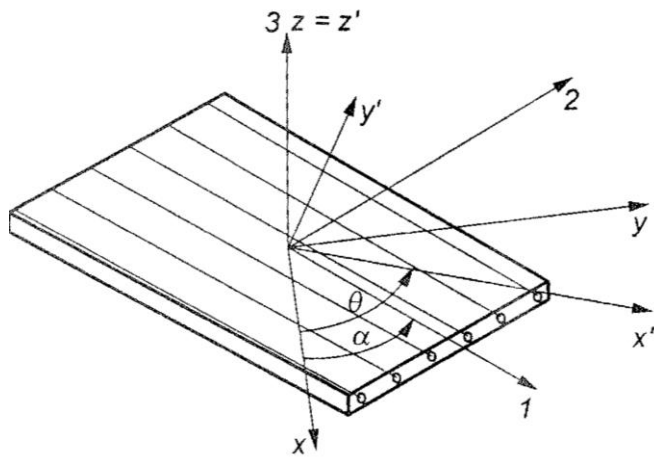
Data acquisition:

E_{11}, E_{22}, E_{33} – tensile tests along the principal orthotropy axes,

G_{12}, G_{23}, G_{31} – shear tests,

$\nu_{ij} = -\frac{\varepsilon_{jj}}{\varepsilon_{ii}}$ – uniaxial tension tests along σ_{ii} directions.

STRAIN AND STRESS IN ORTHOTROPIC LAYER



$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_{11}} & -\frac{\nu_{21}}{E_{22}} & -\frac{\nu_{31}}{E_{33}} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_{11}} & \frac{1}{E_{22}} & -\frac{\nu_{32}}{E_{33}} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_{11}} & -\frac{\nu_{23}}{E_{22}} & \frac{1}{E_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}$$

If a thin layer is loaded in xy-plane along the principal orthotropy direction 1 and/or 2, then:

$$\begin{aligned} \tau_{xz} = \tau_{yz} = \sigma_{zz} &= 0 \\ \tau_{31} = \tau_{23} = \sigma_{33} &= 0 \end{aligned}$$

$$\gamma_{31} = 0, \quad \gamma_{23} = 0, \quad \varepsilon_{33} = -\frac{\nu_{13}}{E_{11}} \sigma_{11} - \frac{\nu_{23}}{E_{22}} \sigma_{22}$$

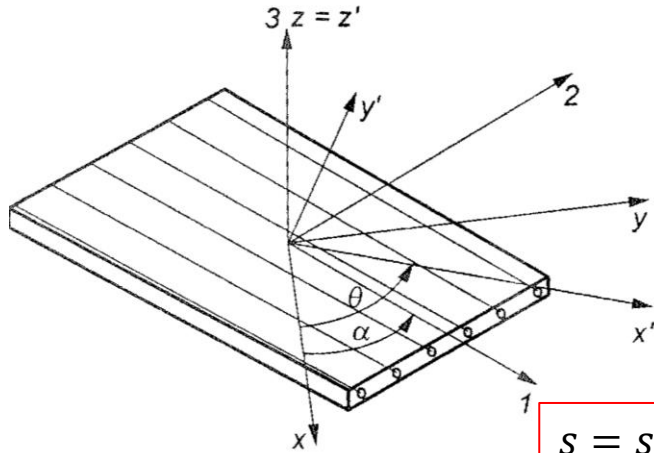
**PLANE
STRESS
CONDITIONS**

Constitutive law for the plane stress in 12-plane can be expressed as:

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_{11}} & -\frac{\nu_{21}}{E_{22}} & 0 \\ -\frac{\nu_{12}}{E_{11}} & \frac{1}{E_{22}} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix} \quad \text{or:} \quad \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \frac{E_{11}}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}} & \frac{E_{22}}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix}$$

There are four independent constants in a constitutive matrix ($E_{11}, E_{22}, \nu_{12}, G_{12}$).

It is usually assumed that $E_{11} > E_{22}$, ν_{12} is called major Poisson's ratio, and ν_{21} - minor Poisson's ratio.



Relationship between strain and stress components in xy- plane can be expressed as:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} S_{11}^* & S_{12}^* & S_{16}^* \\ S_{12}^* & S_{22}^* & S_{26}^* \\ S_{16}^* & S_{26}^* & S_{66}^* \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11}^* & Q_{12}^* & Q_{16}^* \\ Q_{12}^* & Q_{22}^* & Q_{26}^* \\ Q_{16}^* & Q_{26}^* & Q_{66}^* \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

$$s = \sin \alpha, c = \cos \alpha$$

$$\begin{aligned} S_{11}^* &= S_{11}c^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}s^4 \\ S_{12}^* &= S_{12}c^4 + (S_{11} + S_{22} - S_{66})s^2c^2 + S_{12}s^4 \\ S_{22}^* &= S_{22}c^4 + (2S_{12} + S_{66})s^2c^2 + S_{11}s^4 \\ S_{66}^* &= S_{66}c^4 + 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})s^2c^2 + S_{66}s^4 \\ S_{16}^* &= (2S_{11} - 2S_{12} - S_{66})sc^3 - (2S_{22} - 2S_{12} - S_{66})s^3c \\ S_{26}^* &= (2S_{12} + S_{66} - 2S_{22})sc^3 - (2S_{12} + S_{66} - 2S_{11})s^3c \end{aligned}$$

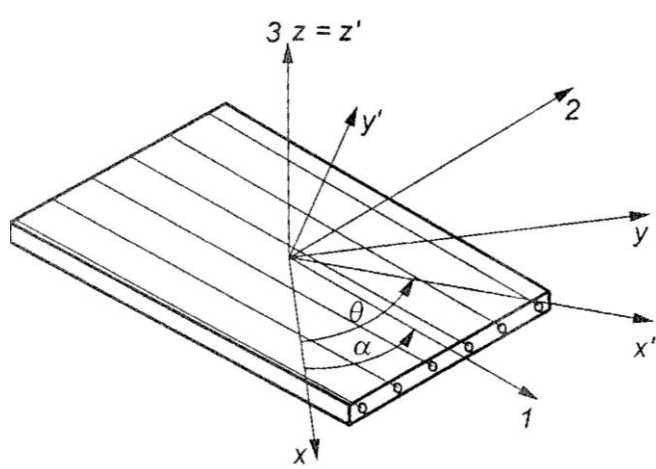
$$\begin{aligned} S_{11} &= \frac{1}{E_{11}} & S_{12} &= \frac{-\nu_{21}}{E_{22}} \\ S_{22} &= \frac{1}{E_{22}} & S_{66} &= \frac{1}{G_{12}} \end{aligned}$$

$$\begin{aligned} Q_{11}^* &= Q_{11}c^4 + (Q_{12} + 2Q_{66})s^2c^2 + Q_{22}s^4 \\ Q_{12}^* &= S_{12}c^4 + (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}s^4 \\ Q_{22}^* &= Q_{22}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2 + Q_{11}s^4 \\ Q_{66}^* &= Q_{66}c^4 + (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}s^4 \\ Q_{16}^* &= (Q_{11} - Q_{12} - 2Q_{66})sc^3 - (Q_{12} - Q_{22} - 2Q_{66})s^3c \\ Q_{26}^* &= (Q_{12} + Q_{22} + 2Q_{66})sc^3 - (Q_{11} + Q_{12} - 2Q_{66})s^3c \end{aligned}$$

$$\begin{aligned} Q_{11} &= \frac{E_{11}}{1 - \nu_{12}\nu_{21}} & Q_{12} &= \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}} \\ Q_{22} &= \frac{E_{22}}{1 - \nu_{12}\nu_{21}} & Q_{66} &= G_{12} \end{aligned}$$

If $\alpha \neq k\frac{\pi}{2}$ the principal stress directions do not coincide with the principal strain directions (full coupling).

Strain and stress components transformation of to $x'y'$ -coordinate system



$$\begin{Bmatrix} \sigma_{x'x'} \\ \sigma_{y'y'} \\ \tau_{x'y'} \end{Bmatrix} = [T] \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_{x'x'} \\ \varepsilon_{y'y'} \\ \frac{\gamma_{x'y'}}{2} \end{Bmatrix} = [T] \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \frac{\gamma_{xy}}{2} \end{Bmatrix}$$

Transformation matrix:

$$[T] = \begin{bmatrix} \cos^2 \theta & \sin^2 \theta & 2 \sin \theta \cos \theta \\ \sin^2 \theta & \cos^2 \theta & -2 \sin \theta \cos \theta \\ -\sin \theta \cos \theta & \sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta \end{bmatrix}$$

FAILURE CRITERIA

In the case of orthotropy, uniaxial stretching at a given load may lead to failure in some directions but not in others. This effect does not occur in isotropic materials. Therefore, orthotropic materials require different **failure criteria**, which are often modifications of those developed for isotropic materials.

Maximum stress criterion

The safe state of the orthotropic layer means that stresses along the principal axes of orthotropy are lower than the experimentally determined limit values X_{ti} , X_{ci} and S_{ij}^w .
(t- tension, c- compression, S- shear, $X_{ci} \neq -X_{ti}$).

$$X_{c1} < \sigma_{11} < X_{t2} ; \quad X_{c2} < \sigma_{22} < X_{t2} ; \quad X_{c3} < \sigma_{33} < X_{t3} \\ |\tau_{12}| < S_{12}^w ; \quad |\tau_{23}| < S_{23}^w ; \quad |\tau_{13}| < S_{31}^w$$

The criterion of maximum stress can be expressed as:

$$k = \max \left\{ \frac{\sigma_{11}}{X_{t1}} \text{ or } \frac{\sigma_{11}}{X_{c1}}, \quad \frac{\sigma_{22}}{X_{t2}} \text{ or } \frac{\sigma_{22}}{X_{c2}}, \quad \frac{\sigma_{33}}{X_{t3}} \text{ or } \frac{\sigma_{33}}{X_{c3}}, \quad \frac{|\tau_{12}|}{S_{12}^w}, \quad \frac{|\tau_{23}|}{S_{23}^w}, \quad \frac{|\tau_{13}|}{S_{31}^w} \right\}$$

($k=0$ – no stress, $k=1$ – failure)

Maximum strain criterion

The safe state of the orthotropic layer means that strains along the principal axes of orthotropy are lower than the experimentally determined limit strain values ε_{ti} , ε_{ci} (t- tension, c- compression), and γ_{ij}^* - shear:

$$\varepsilon_{c1} < \varepsilon_{11} < \varepsilon_{t2}, \quad \varepsilon_{c2} < \varepsilon_{22} < \varepsilon_{t2}, \quad \varepsilon_{c3} < \varepsilon_{33} < \varepsilon_{t3}$$
$$|\gamma_{12}| < \gamma_{12}^*, \quad |\gamma_{23}| < \gamma_{23}^*, \quad |\gamma_{31}| < \gamma_{31}^*$$

The criterion of maximum strain can be expressed as a single factor:

$$k = \max \left\{ \frac{\varepsilon_{11}}{\varepsilon_{t1}} \text{ or } \frac{\varepsilon_{11}}{\varepsilon_{c1}}, \quad \frac{\varepsilon_{22}}{\varepsilon_{t2}} \text{ or } \frac{\varepsilon_{22}}{\varepsilon_{c2}}, \quad \frac{\varepsilon_{33}}{\varepsilon_{t3}} \text{ or } \frac{\varepsilon_{33}}{\varepsilon_{c3}}, \quad \frac{|\gamma_{12}|}{\gamma_{12}^*}, \quad \frac{|\gamma_{23}|}{\gamma_{23}^*}, \quad \frac{|\gamma_{13}|}{\gamma_{13}^*} \right\}$$

(k= 0 – no stress, k= 1 – failure)

Tsai–Wu criterion

This criterion includes contribution of all stress components and assumes that the material effort represents a polynomial function of stress. The safe state of the orthotropic layer is inside the ellipse:

$$F_1\sigma_{11} + F_2\sigma_{22} + F_6\tau_{12} + F_{11}\sigma_{11}^2 + F_{22}\sigma_{22}^2 + F_{66}\tau_{12}^2 + 2F_{12}\sigma_{11}\sigma_{22} = 1$$

; where material constants: $F_1, F_2, F_6, F_{11}, F_{22}, F_{66}, F_{12}$ depend on the limit values: $X_{c1}, X_{t1}, X_{c2}, X_{t2}, S^w_{12}$:

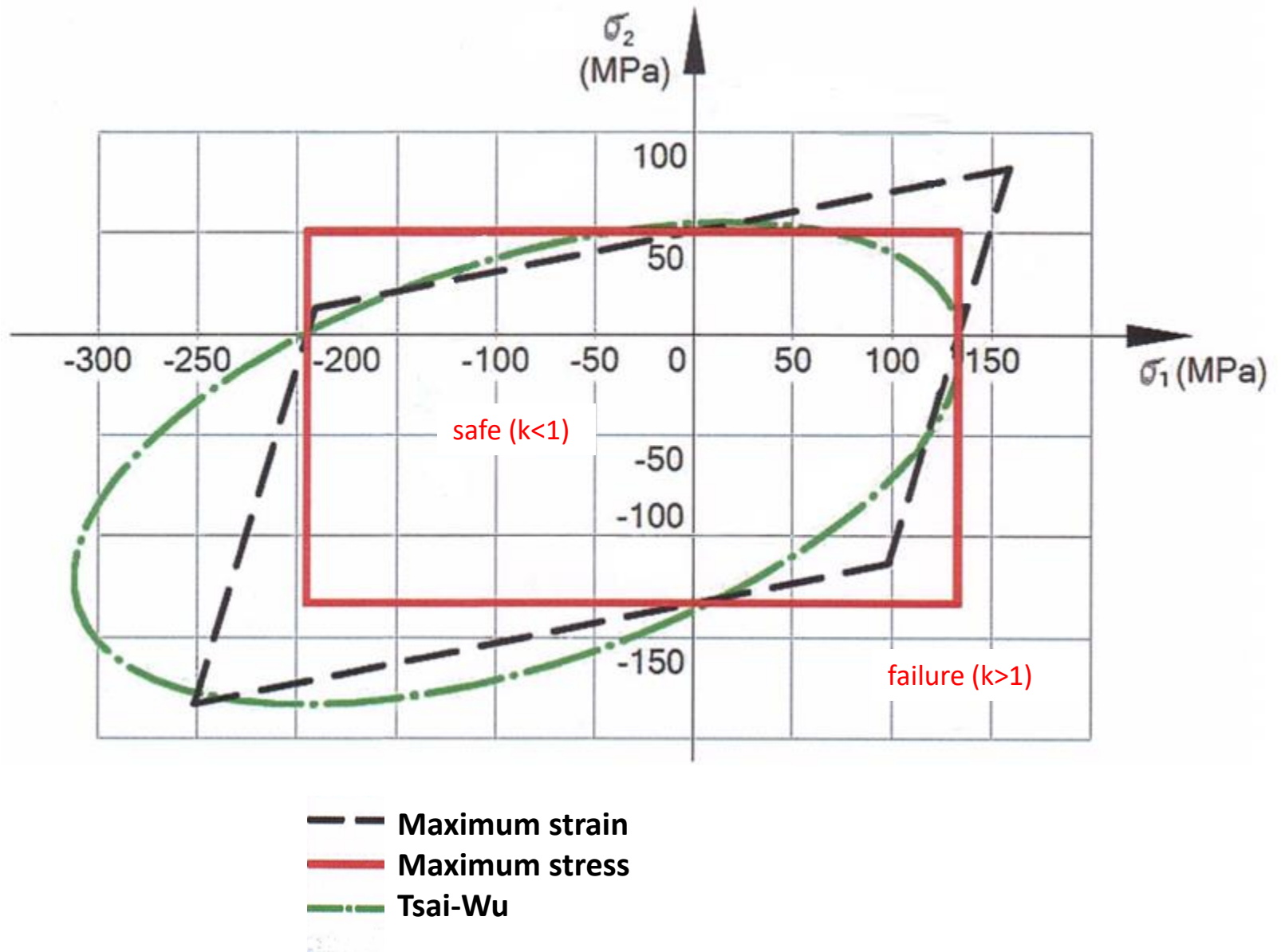
$$F_1 = \frac{1}{X_{t1}} + \frac{1}{X_{c1}}, \quad F_{11} = \frac{-1}{X_{c1}X_{t1}}$$

$$F_2 = \frac{1}{X_{t2}} + \frac{1}{X_{c2}}, \quad F_{22} = \frac{-1}{X_{t2}X_{c2}}$$

$$F_{66} = \frac{1}{S_{12}^w}, \quad F_6 = 0$$

$$F_{12} = -\frac{\sqrt{F_{11} \cdot F_{22}}}{2}$$

Comparison of Stress, Strain, and Tsai–Wu criteria



Example 1. Unidirectional stretching of a single orthotropic layer

Material properties: $E_{11} = 2 \cdot 10^5 \text{ MPa}$, $E_{22} = 0,5 \cdot 10^5 \text{ MPa}$, $\nu_{12} = 0,25$, $G_{12} = 2 \cdot 10^4 \text{ MPa}$. The layer is loaded by $\sigma_{xx} = 10 \text{ MPa}$. Find stress in xy-coordinate system as a function of α angle.

Analytical solution:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} S_{11}^* & S_{12}^* & S_{16}^* \\ S_{12}^* & S_{22}^* & S_{26}^* \\ S_{16}^* & S_{26}^* & S_{66}^* \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ 0 \\ 0 \end{Bmatrix},$$

yields:

$$\begin{aligned} \varepsilon_{xx} &= S_{11}^* \sigma_{xx}, \\ \varepsilon_{yy} &= S_{12}^* \sigma_{xx}, \\ \gamma_{xy} &= S_{16}^* \sigma_{xx}, \end{aligned}$$

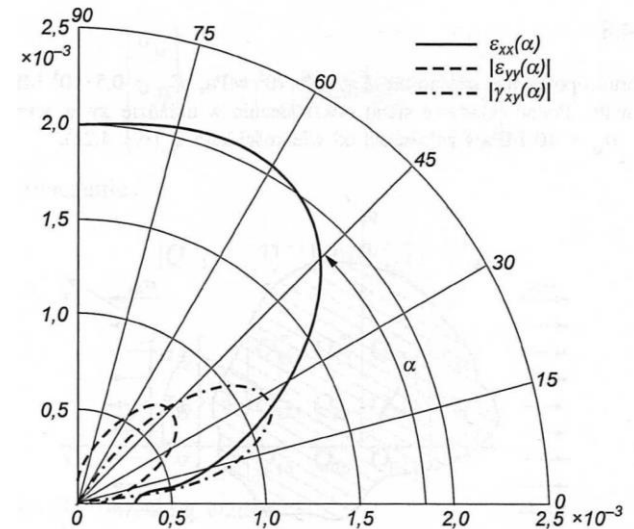
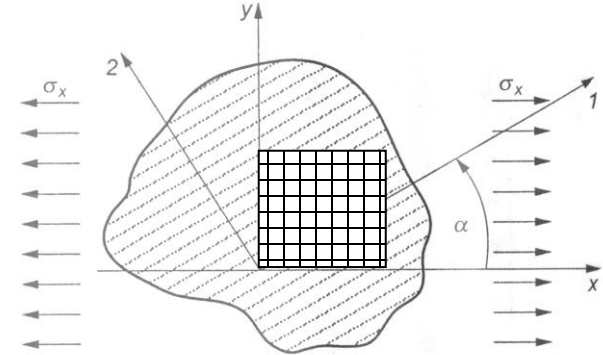
where:

$$\begin{aligned} S_{11}^* &= S_{11} c^4 + (2S_{12} + S_{66}) s^2 c^2 + S_{22} s^4, \\ S_{12}^* &= S_{12} c^4 + (S_{11} + S_{22} - S_{66}) s^2 c^2 + S_{12} s^4, \\ S_{16}^* &= (2S_{12} + S_{66} - 2S_{22}) s c^3 - (2S_{12} + S_{66} - 2S_{11}) s^3 c. \end{aligned}$$

$$S_{11} = \frac{1}{E_{11}}, \quad S_{22} = \frac{1}{E_{22}}, \quad S_{12} = \frac{-\nu_{21}}{E_{22}} = \frac{-\nu_{12}}{E_{11}}, \quad S_{66} = G_{12}.$$

Strain components for a set of α values

α	0	5	15	30	45	60	75	85	90
$10^4 \varepsilon_{xx}$	0,5	0,528	0,741	1,297	1,812	2,047	2,040	2,006	2,000
$10^4 \varepsilon_{yy}$	-0,125	-0,142	-0,266	-0,547	-0,688	-0,547	-0,266	-0,142	-0,125
$10^4 \gamma_{xy}$	0	-0,323	-0,862	-1,137	-0,75	-0,162	-0,112	-0,062	0



DOF Solution (ANSYS)

Linear Orthotropic Properties for Material Number 1

Linear Orthotropic Material Properties for Material Number

Choose Poisson's Ratio

T1

Temperatures	
EX	2E+05
EY	50000
EZ	50000
PRXY	0.25
PRYZ	0.3
PRXZ	0.25
GXY	20000
GYZ	15000
GXZ	20000

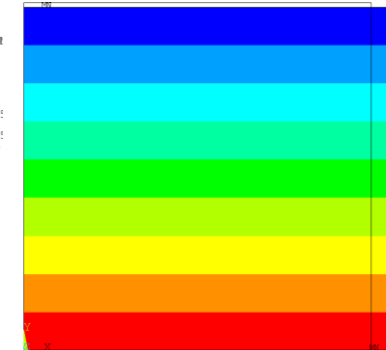
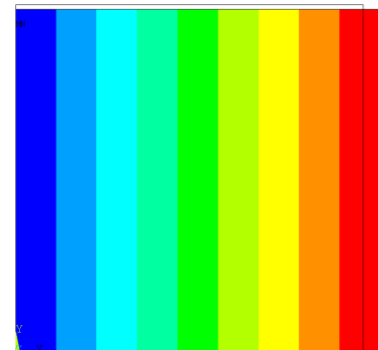
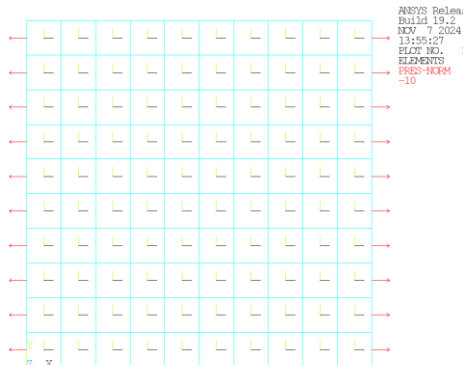
Add Temperature Delete Temperature

OK Cancel

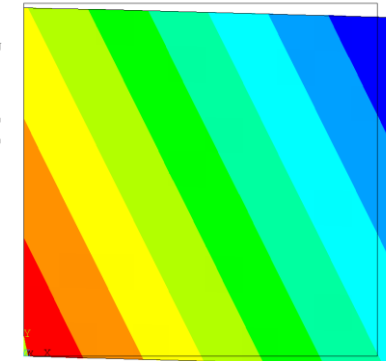
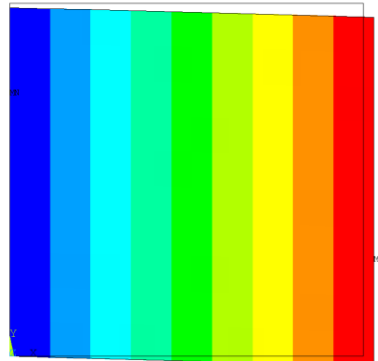
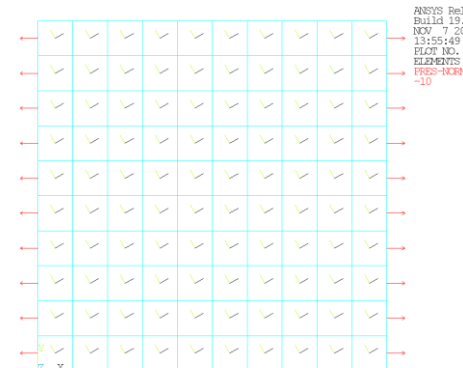
UX [mm]

UY [mm]

$\alpha = 0^\circ$



$\alpha = 30^\circ$

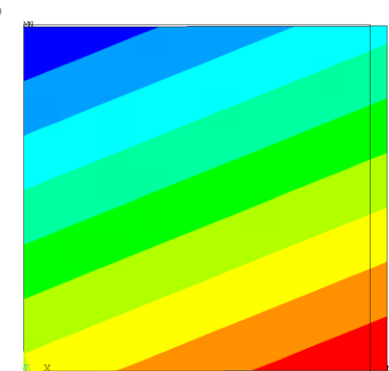
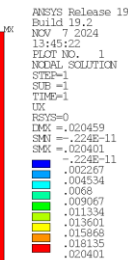
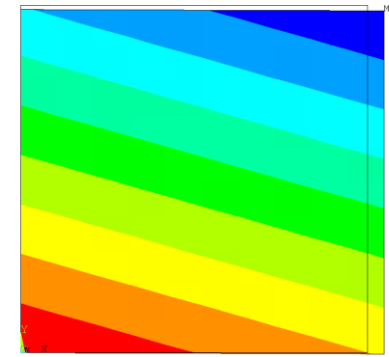
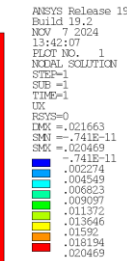
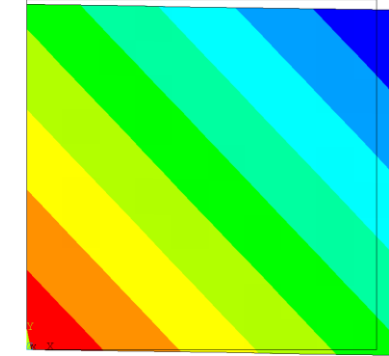
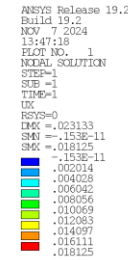
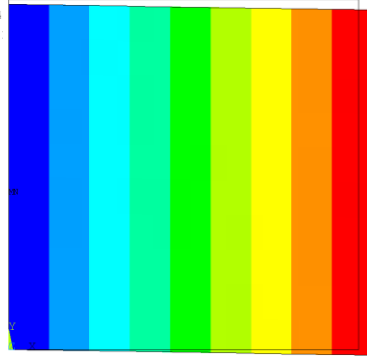
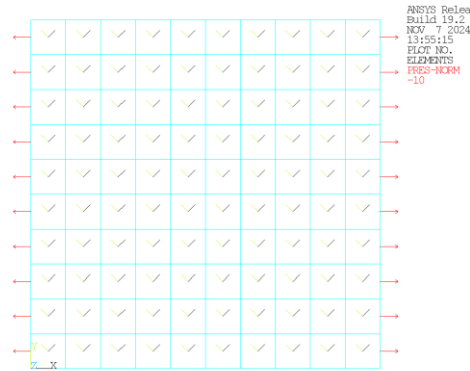


DOF Solution in ANSYS

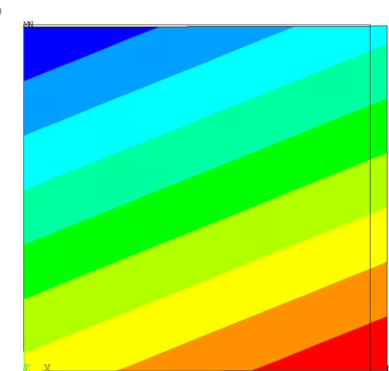
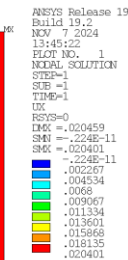
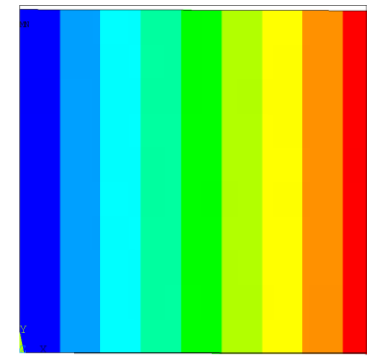
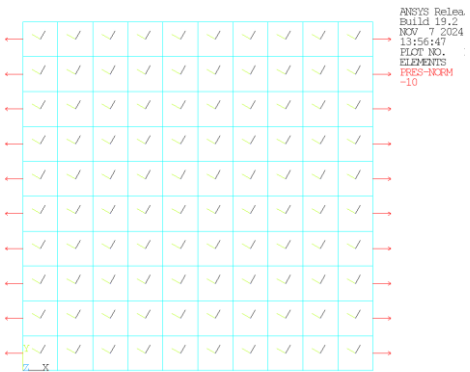
UX [mm]

UY [mm]

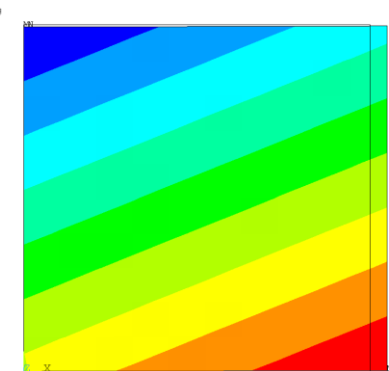
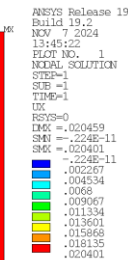
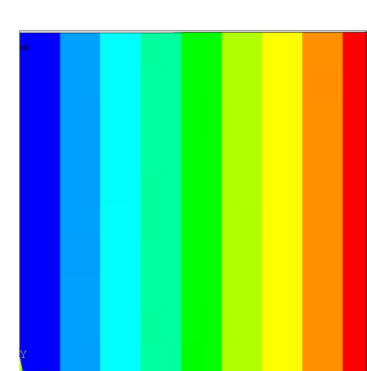
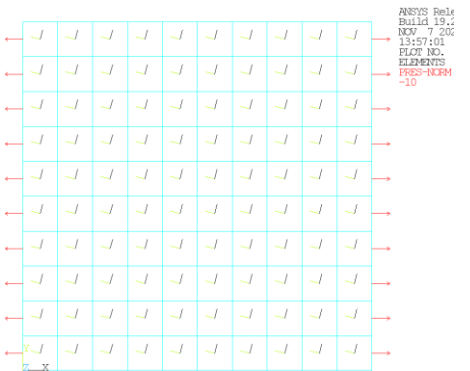
$\alpha = 45^\circ$



$\alpha = 60^\circ$



$\alpha = 75^\circ$



Strain

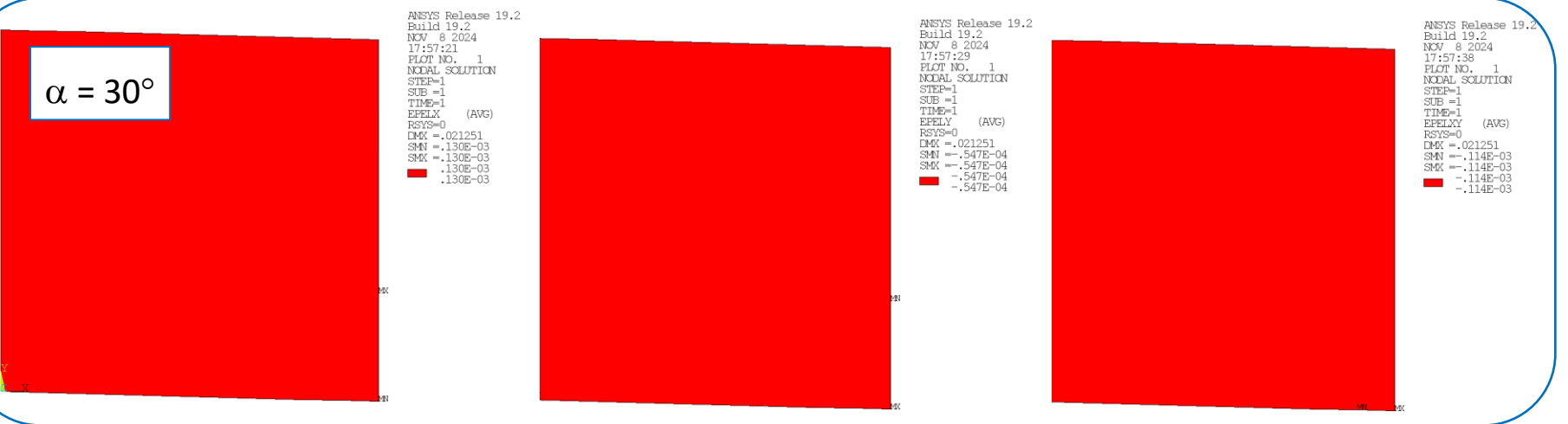
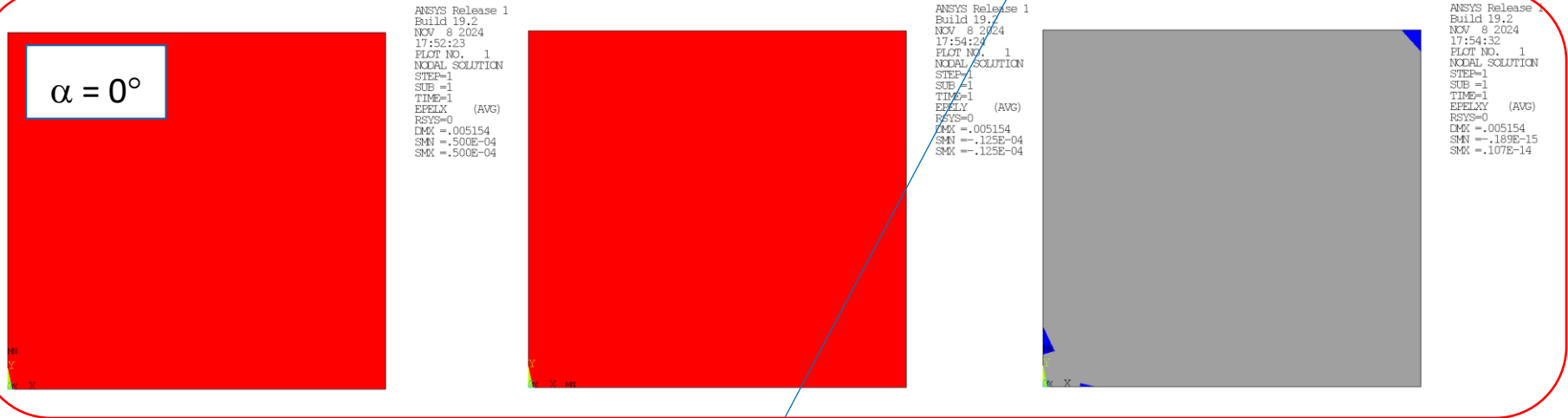
Nodal solution in ANSYS

α	0	5	15	30	45	60	75	85	90
$10^4 \varepsilon_{xx}$	0,5	0,528	0,741	1,297	1,812	2,047	2,040	2,006	2,000
$10^4 \varepsilon_{yy}$	-0,125	-0,142	-0,266	-0,547	-0,688	-0,547	-0,266	-0,142	-0,125
$10^4 \gamma_{xy}$	0	-0,323	-0,862	-1,137	-0,75	-0,162	-0,112	-0,062	0

EPELX (ε_{xx})

EPELY (ε_{yy})

EPELXY (γ_{xy})



Strain

Nodal solution in ANSYS

α	0	5	15	30	45	60	75	85	90
$10^4 \varepsilon_{xx}$	0,5	0,528	0,741	1,297	1,812	2,047	2,040	2,006	2,000
$10^4 \varepsilon_{yy}$	-0,125	-0,142	-0,266	-0,547	-0,688	-0,547	-0,266	-0,142	-0,125
$10^4 \gamma_{xy}$	0	-0,323	-0,862	-1.137	-0,75	-0,162	-0,112	-0,062	0

EPELX (ε_{xx})

EPELY (ε_{yy})

EPELXY (γ_{xy})

$\alpha = 45^\circ$

ANSYS Release 19.2
Build 19.2
NOV 8 2024
18:04:07
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
EPELX (AVG)
RSYS=0
DMX =.023133
SMN =.181E-03
SMX =.181E-03
MIN =.181E-03
MAX =.181E-03

ANSYS Release 19.2
Build 19.2
NOV 8 2024
18:04:14
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
EPELY (AVG)
RSYS=0
DMX =.023133
SMN =.688E-04
SMX =.687E-04
MIN =.688E-04
MAX =.687E-04

ANSYS Release 19.2
Build 19.2
NOV 8 2024
18:04:42
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
EPELXY (AVG)
RSYS=0
DMX =.023133
SMN =.750E-04
SMX =.750E-04
MIN =.750E-04
MAX =.750E-04

$\alpha = 60^\circ$

ANSYS Release 19.2
Build 19.2
NOV 8 2024
18:07:21
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
EPELX (AVG)
RSYS=0
DMX =.021663
SMN =.205E-04
SMX =.205E-04
MIN =.205E-04
MAX =.205E-04

ANSYS Release 19.2
Build 19.2
NOV 8 2024
18:07:28
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
EPELY (AVG)
RSYS=0
DMX =.021663
SMN =.547E-04
SMX =.547E-04
MIN =.547E-04
MAX =.547E-04

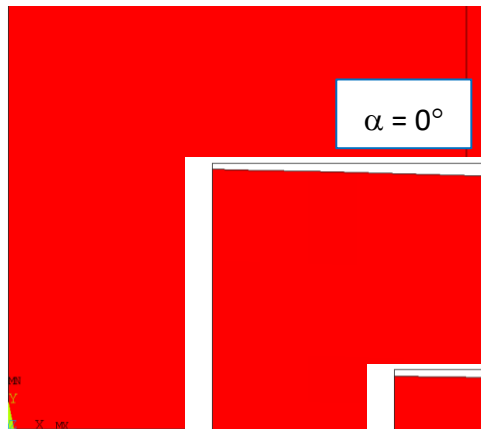
ANSYS Release 19.2
Build 19.2
NOV 8 2024
18:07:39
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
EPELXY (AVG)
RSYS=0
DMX =.021663
SMN =.162E-04
SMX =.162E-04
MIN =.162E-04
MAX =.162E-04

Maximum stress criterion



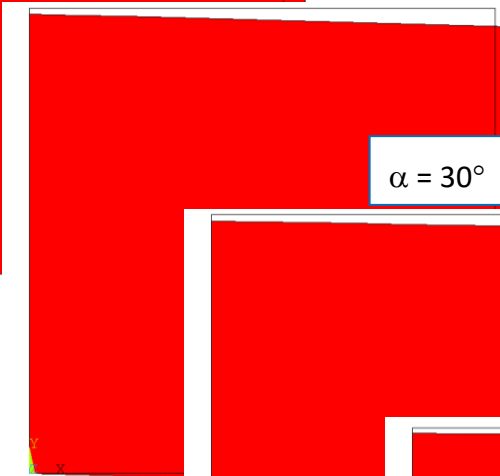
For orthotropic materials, equivalent stress criteria (Huber, Tresca) provide erroneous assessment of failure.

Tsai Wu criterion



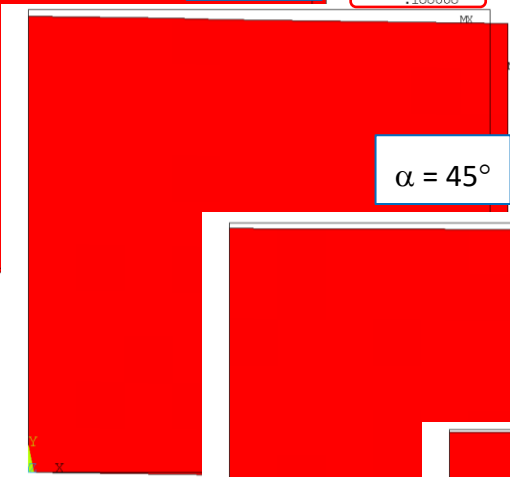
```

NODAL SOLUTION
STEP=1
SUB =1
TIME=1
FAILTWSI (AVG)
RSYS=0
DMX =.005154
SMN =.625E-03
SMX =.625E-03
    
```



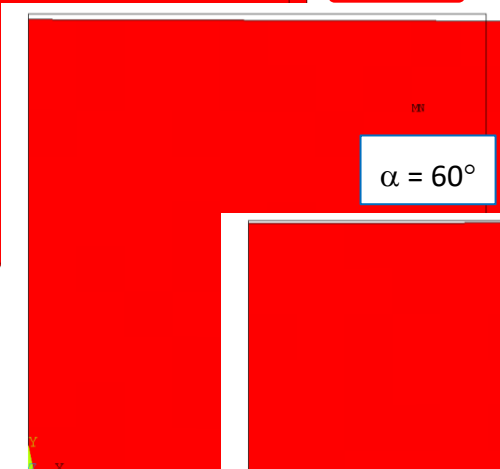
```

Build 19.2
NOV 8 2024
19:53:51
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
FAILTWSI (AVG)
RSYS=0
DMX =.021251
SMN =.188008
SMX =.188008
    
```



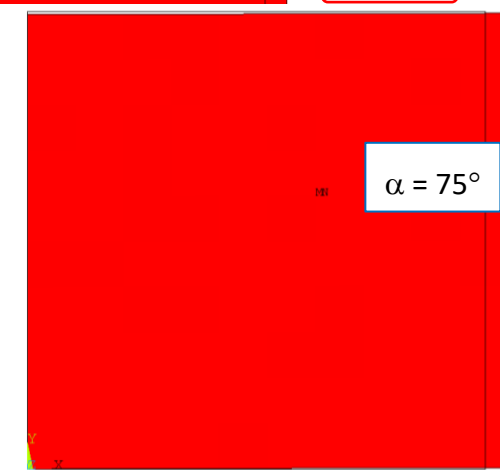
```

Build 19.2
NOV 8 2024
19:53:36
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
FAILTWSI (AVG)
RSYS=0
DMX =.023133
SMN =.252031
SMX =.252031
    
```



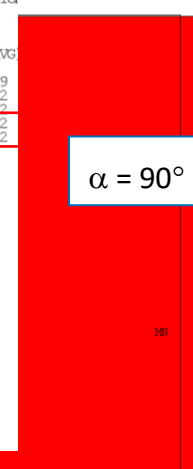
```

Build 19.2
NOV 8 2024
19:53:19
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
FAILTWSI (AVG)
RSYS=0
DMX =.021663
SMN =.192695
SMX =.192695
    
```



```

Build 19.2
NOV 8 2024
19:53:05
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
FAILTWSI (AVG)
RSYS=0
DMX =.020459
SMN =.071052
SMX =.071052
    
```



```

ANSYS Release
Build 19.2
NOV 8 2024
19:52:49
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
FAILTWSI (P)
RSYS=0
DMX =.02003
SMN =.01
SMX =.01
    
```

- General Postproc
- Data & File Opts
- Results Summary
- Read Results
- Failure Criteria
 - Temp Variation
 - Add/Edit
 - Delete
 - List
 - Criteria Check
- Plot Results
- List Results
- Query Results
- Options for Outp
- Results Viewer
- Nodal Calcs
- Element Table
- Path Operations
- Surface Operations
- Load Case
- Check Elem Shape
- Write Results
- ROM Operations
- Submodeling
- Safety Factor
- Define/Modify
- Nonlinear Diagnostics
- Reset
- Manual Rezoning
- TimeHist Postpro
- ROM Tool
- Radiation Opt
- Session Editor

Add/Edit Failure Criteria

[FC] Temperature-independent failure criteria for material 1

	X	Y	Z
Strain in Tension	0.002	0.002	0.002
Strain in Compression	-0.002	-0.002	-0.002
Strain in Shear	0.0025	0.001	0.001
Stress in Tension	400	100	1
Stress in Compression	-400	-100	-1
Stress in Shear	10	1	1
Stress Coupling Coefficients	-1	-1	-1

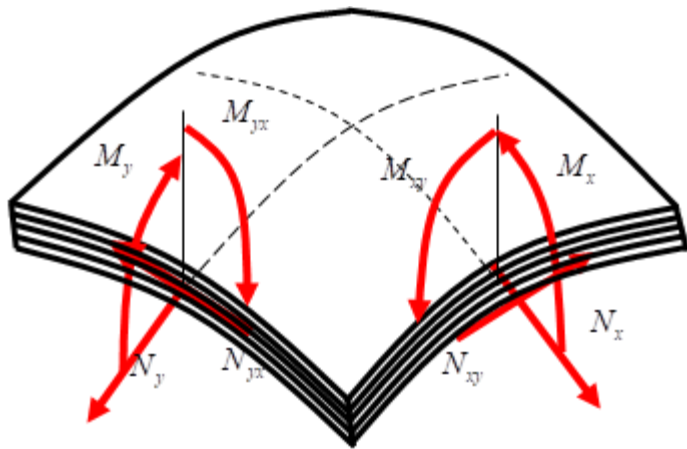
OK Cancel Help

For orthotropic materials, equivalent stress criteria (Huber, Tresca) provide erroneous assessment of failure.

BASICS OF MULTILAYER COMPOSITES

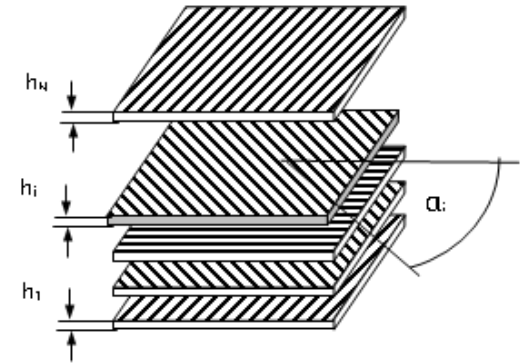
Assumptions:

- the laminate is composed of multiple orthotropic layers tightly bonded together,
- stresses perpendicular to the layers can be ignored for thin laminates (thickness $\ll$ length and width),
- strain components are continuous at the interface between layers,
- stresses discontinuities are allowable due to differences in material properties,
- small deformations,
- the line normal to the laminate surface remains normal to it after deformation (Kirchoff's hypothesis)



Internal forces in a multilayer composite

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix}$$



where:

$\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0$ – strain components in the middle layer,

$\kappa_x = \frac{\partial^2 w_0}{\partial x^2}, \kappa_y = \frac{\partial^2 w_0}{\partial y^2}, \kappa_{xy} = \frac{\partial^2 w_0}{\partial x \partial y}$ – curvatures of the middle layer,

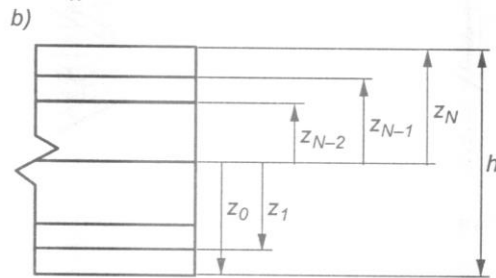
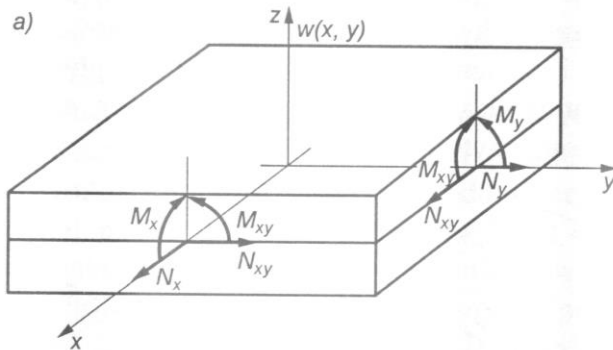
w_0 – deflection of the middle layer,

$N_x, N_y, N_{xy}, M_x, M_y, M_{xy}$ – internal forces,

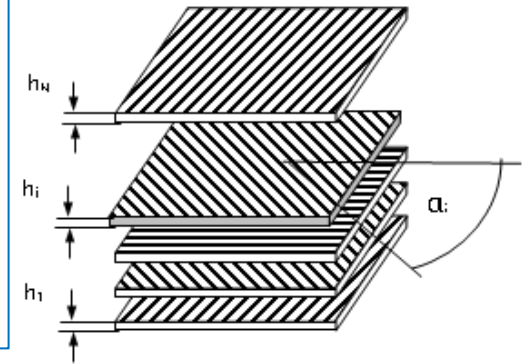
A – in-plane laminate stiffness matrix, **B** – coupling stiffness matrix,

D – flexural-torsional stiffness matrix.

BASICS OF MULTILAYER COMPOSITES



$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix}$$



Matrices **A**, **B**, **D** (3×3), are symmetric and depend on material properties of layers, their orientation and location.

$$A_{ij} = \sum_{k=1}^N Q_{ij}^{*k} (z_k - z_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^N Q_{ij}^{*k} (z_k^2 - z_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^N Q_{ij}^{*k} (z_k^3 - z_{k-1}^3)$$

where: z_k – distance of k-layer from the middle layer

Q_{ij}^{*k} – stiffness matrix of k-layer transformed to xy-coordinate system

Example 2. Unidirectional stretching of a six-layer laminate

Thickness of each layer is 0.125 mm, reinforcement directions are $[45^\circ, -45^\circ, 45^\circ, -45^\circ, 45^\circ, -45^\circ]$. Size: $L_x = 100$ mm, $L_y = 100$ mm. A carbon-epoxy composite has Young's moduli: $E_{11} = 211000$ MPa, $E_{22} = E_{33} = 5300$ MPa, Kirchoff's moduli $G_{12} = G_{31} = 2600$ MPa, $G_{23} = 2000$ MPa. Poisson's ratios: $\nu_{12} = \nu_{31} = 0.25$, $\nu_{23} = 0.2$. Load is applied as uniform traction $N_x = 10$ N/mm. Find deformation and stresses in layers.

Analytical solution:

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} 1.348e-4 & -1.216e-4 & 0 & 0 & 0 & -1.69e-5 \\ -1.216e-4 & 1.348e-4 & 0 & 0 & 0 & -1.69e-5 \\ 0 & 0 & 2.685e-5 & -1.69e-5 & -1.69e-5 & 0 \\ 0 & 0 & -1.69e-5 & 2.876e-3 & -2.594e-3 & 0 \\ 0 & 0 & -1.69e-5 & -2.594e-3 & 2.876e-3 & 0 \\ -1.69e-5 & -1.69e-5 & 0 & 0 & 0 & 5.728e-4 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Strain in the middle layer:

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} 1.348e-3 \\ -1.216e-3 \\ 0 \\ 0 \\ 0 \\ -1.69e-4 \end{bmatrix}$$

Displacements (assuming small deformations):

$$\begin{aligned} U_x &= \varepsilon_x^0 L_x = 0,135 \text{ mm}, \\ U_y &= \varepsilon_y^0 L_y = -0,122 \text{ mm}, \\ U_z &= \kappa_{xy} \frac{L_x L_y}{2} = -0,845 \text{ mm}. \end{aligned}$$

FE model of a six-layer laminate

Analytical solution:

$$U_x = \epsilon_x^0 L_x = 0,135 \text{ mm},$$

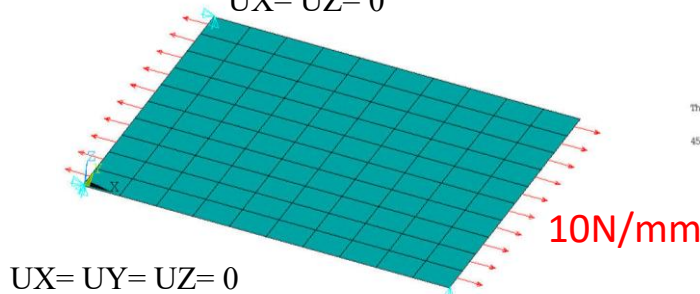
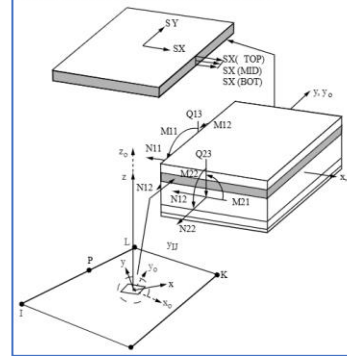
$$U_y = \epsilon_y^0 L_y = -0,122 \text{ mm},$$

$$U_z = \kappa_{xy} \frac{L_x L_y}{2} = -0,845 \text{ mm}.$$

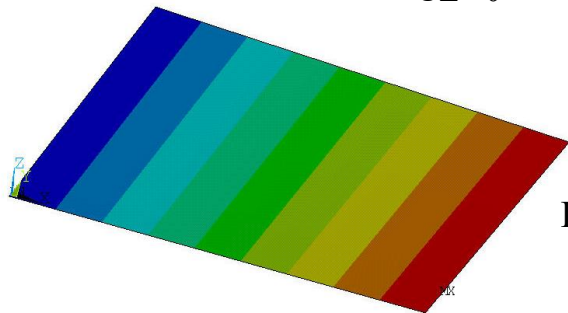
$$U_x = U_z = 0$$

$$\begin{bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} 1.348e-3 \\ -1.216e-3 \\ 0 \\ 0 \\ 0 \\ -1.69e-4 \end{bmatrix}$$

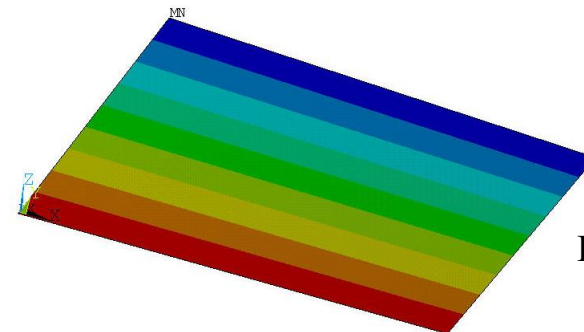
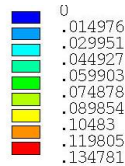
Figure 281.2: SHELL281 Stress Output



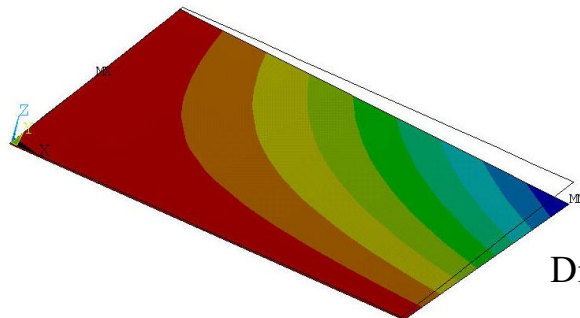
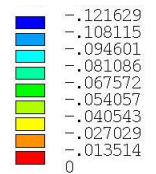
$$U_z = 0$$



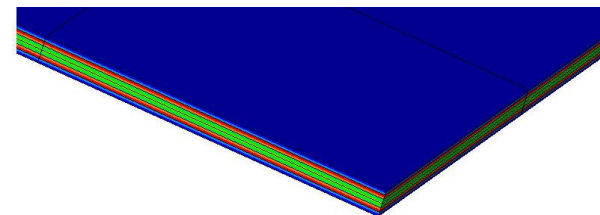
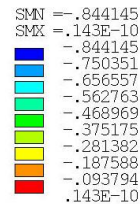
Displacement UX (mm)



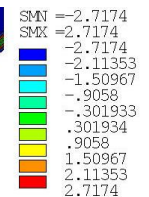
Displacement UY (mm)



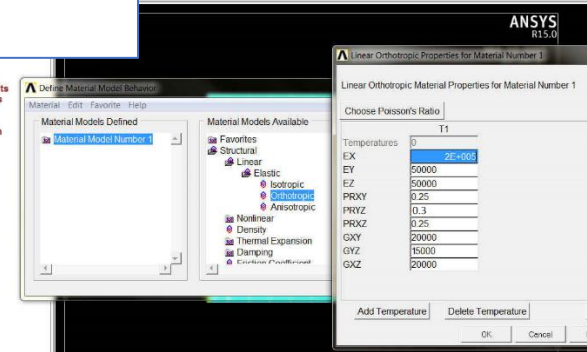
Displacement UZ (mm)



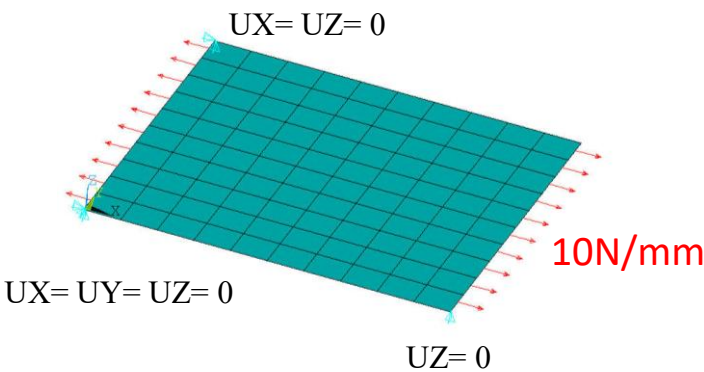
Stress SY in layers (σ_y)



Material Props
Material Library
Temperature Units
Electromag Units
Material Models
Convert ALPx
Change Mat Num
Write to File
Read from File
Sections
Modeling
Meshing
Checking Ctrls
Numbering Ctrls
Archive Model
Coupling / Ceqn
Multi-field Set Up
Loads
Physics
Path Operations
Solution
General Postproc
TimeHist Postpro
RDM Tool
Prob Design
Radiation Opt



Example 3. Unidirectional stretching of a four-layer laminate



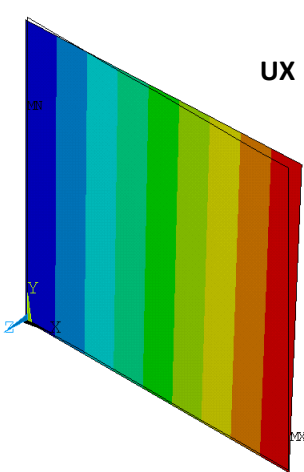
The limit values of stress and strain

Add/Edit Failure Criteria					
[FC] Temperature-independent failure criteria for material 1					
Strain in Tension		X	Y	Z	
					0.0019 0.0019 0.001
Strain in Compression		X	Y	Z	
					-0.0019 -0.0019 -0.001
Strain in Shear		XY	YZ	XZ	
					0.0025 0.001 0.001
Stress in Tension		X	Y	Z	
					400 10 1
Stress in Compression		X	Y	Z	
					-400 -10 -1
Stress in Shear		XY	YZ	XZ	
					10 1 1
Stress Coupling Coefficients		XY	YZ	XZ	
					-1 -1 -1

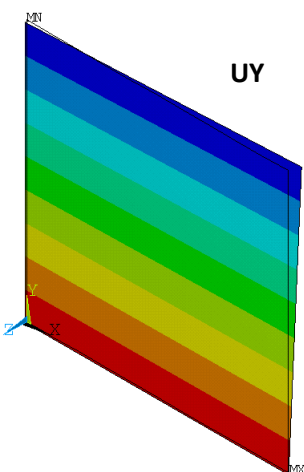
Material properties

Linear Orthotropic Properties for Material Num1	
Linear Orthotropic Material Properties for	
Choose Poisson's Ratio	
Temperatures T1	
EX	2.11E+05
EY	5300
EZ	1000
PRXY	0.25
PRYZ	0
PRXZ	0
GXY	2600
GYZ	1000
GXZ	1000
Add Temperature Delete Temp	

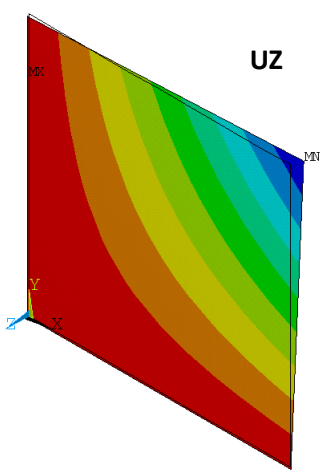
DOF Solution



21:33:21 1
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.804673
SMN =-.716E-12
SMX =.101645
-.716E-12
.011294
.022588
.033882
.045176
.05647
.067764
.079057
.090351
.101645



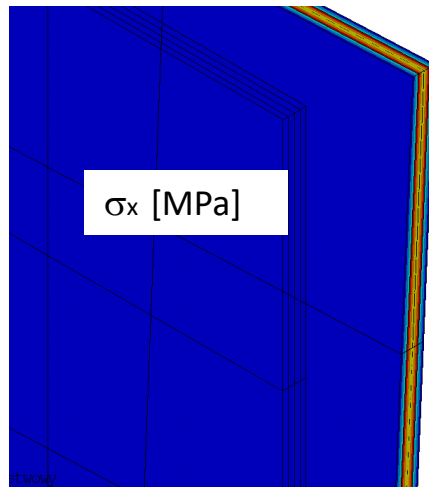
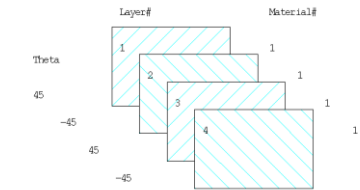
21:33:29 1
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.804673
SMN =-.090662
SMX =.562E-12
-.090662
-.080589
-.070515
-.060442
-.050368
-.040294
-.030221
-.020147
-.010074
.562E-12



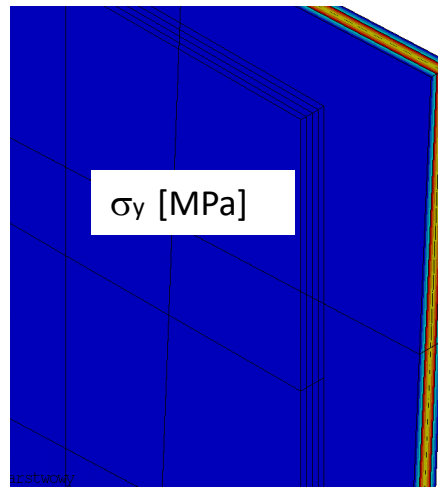
21:33:37 1
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UZ (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.804673
SMN =-.793062
SMX =.363E-11
-.793062
-.704944
-.616826
-.528708
-.44059
-.352472
-.264354
-.176236
-.088118
.363E-11

Stress components in layers (two types of reinforcement)

[45°, -45°, 45°, -45°] – asymmetrical



NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.804769
SMN =6.93645
SMX =13.0636
6.93645
7.61724
8.29803
8.97882
9.65961
10.3404
11.0212
11.702
12.3828
13.0636

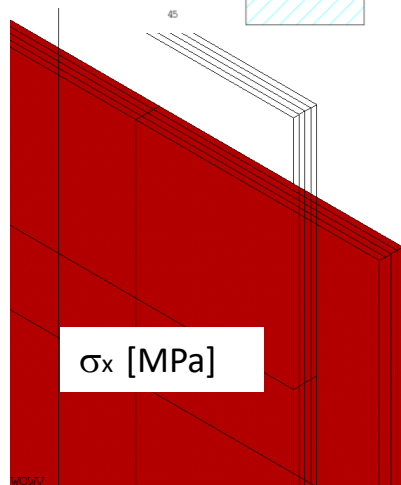
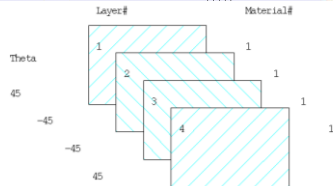


NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.804769
SMN =-3.06355
SMX =3.06355
-3.06355
-2.38276
-1.70197
-1.02118
-.340395
.340395
1.02118
1.70197
2.38276
3.06355

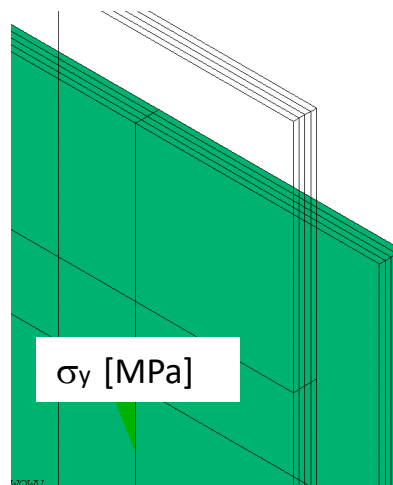


NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SXY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.804769
SMN =-7.77811
SMX =7.77811
-7.77811
-6.04964
-4.32117
-2.5927
-.864234
.864234
2.5927
4.32117
6.04964
7.77811

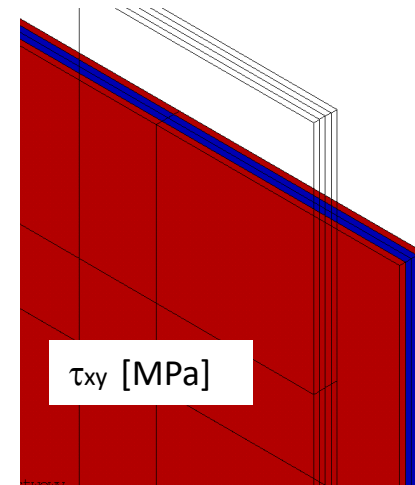
[45°, -45°, -45°, 45°] – symmetrical



NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.136134
SMN =-1.36134
SMX =1.36134
-1.36134
-.566E-09
-.836E-09
-.566E-09
.573E-10
.213E-09
.369E-09
.525E-09
.681E-09
.836E-09

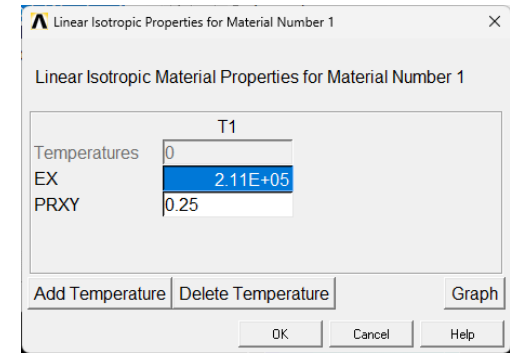


NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.136134
SMN =-.566E-09
SMX =.836E-09
-4.10E-09
-.254E-09
-.985E-10
.573E-10
.213E-09
.369E-09
.525E-09
.681E-09
.836E-09

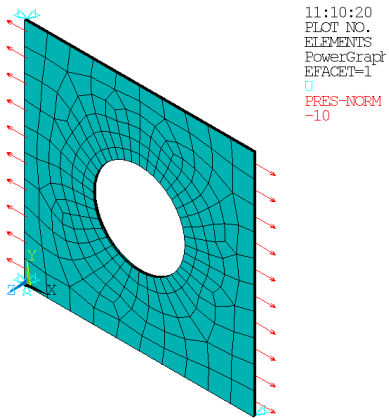


NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SXY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.136134
SMN =-4.69742
SMX =4.69742
-4.69742
-3.65355
-2.60968
-1.56581
-.521936
.521936
1.56581
2.60968
3.65355
4.69742

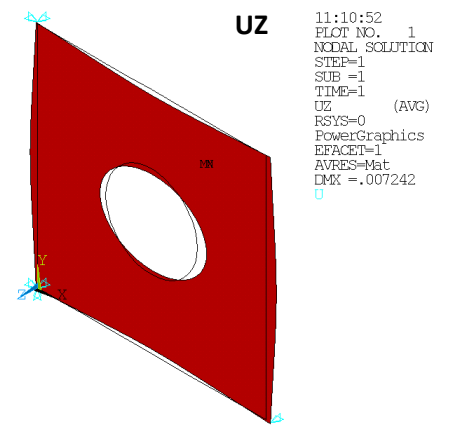
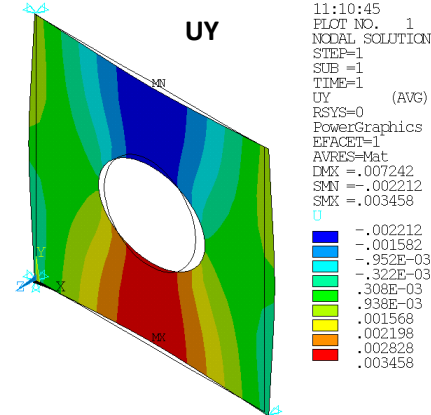
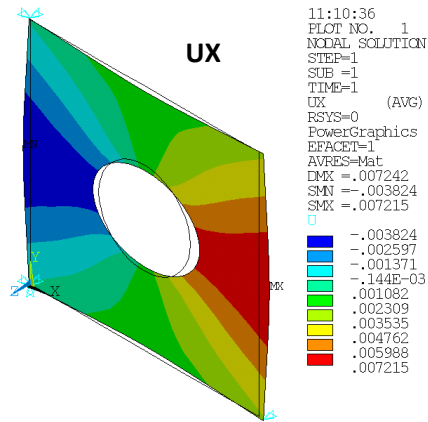
Example 4a. An isotropic plate with a hole ($\phi = 40\text{mm}$)



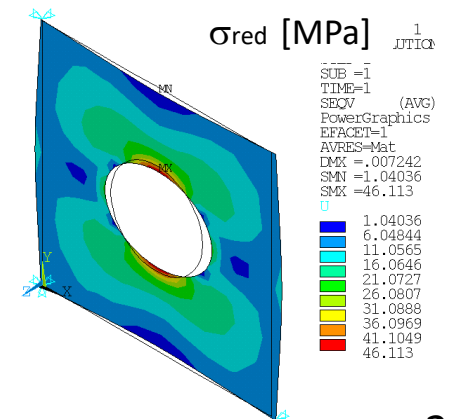
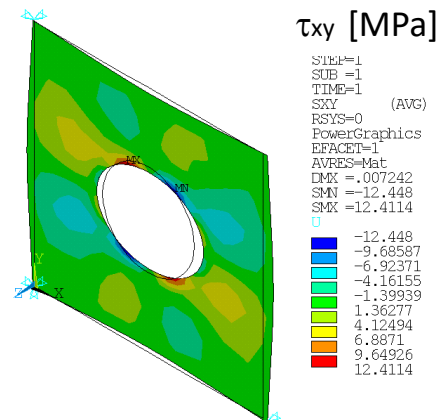
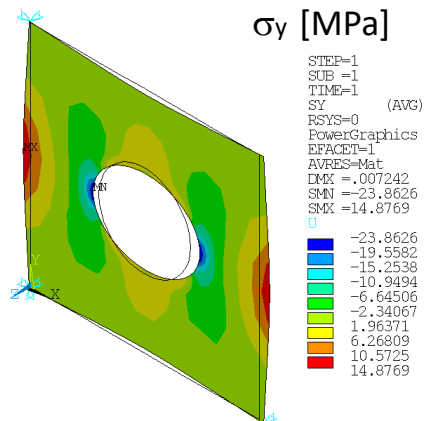
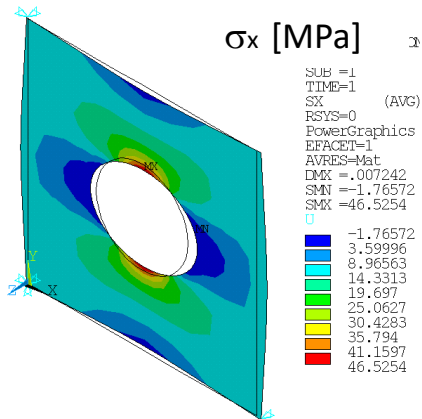
FE model



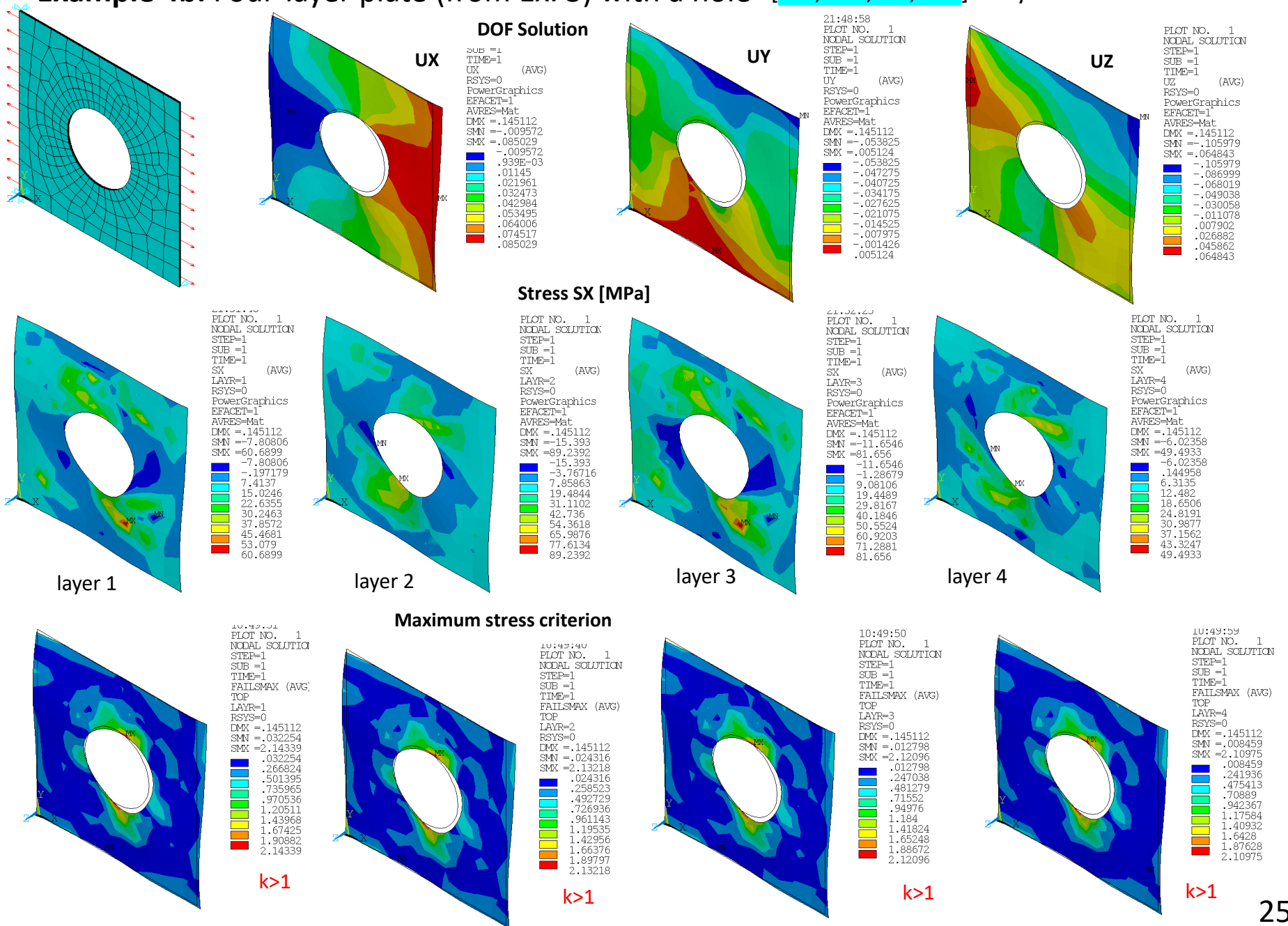
DOF Solution



Stress

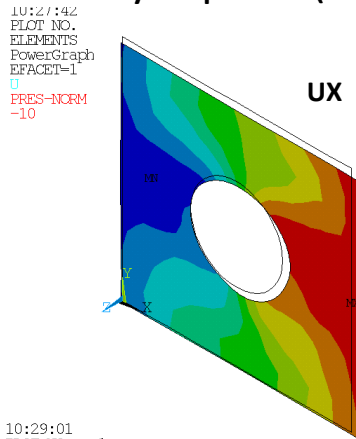
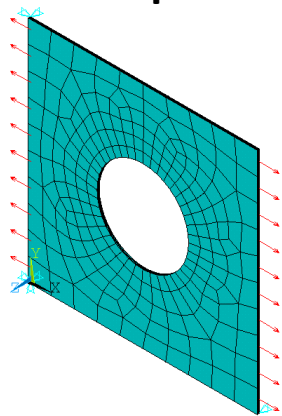


Example 4b. Four-layer plate (from Ex. 3) with a hole [45°, -45°, 45°, -45°] – asymmetric reinforcement

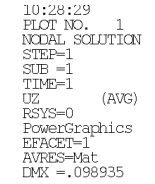
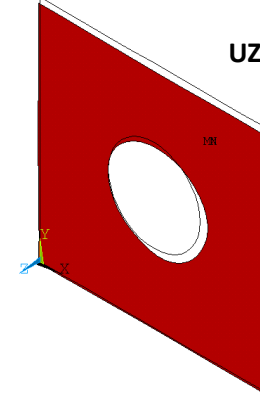
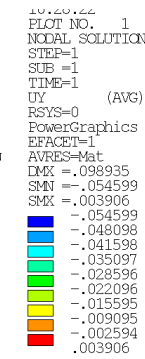
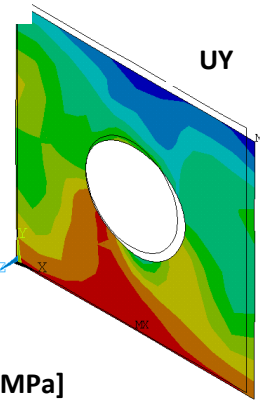
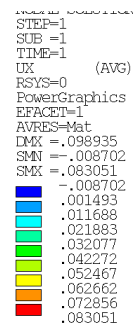


Example 4c. Four-layer plate (from Ex. 3) with a hole

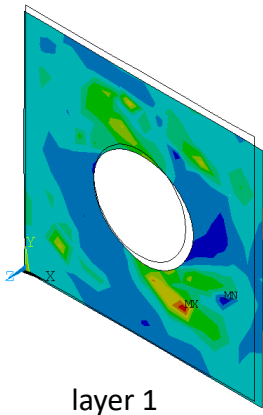
[45°, -45°, -45°, 45°] – symmetric reinforcement



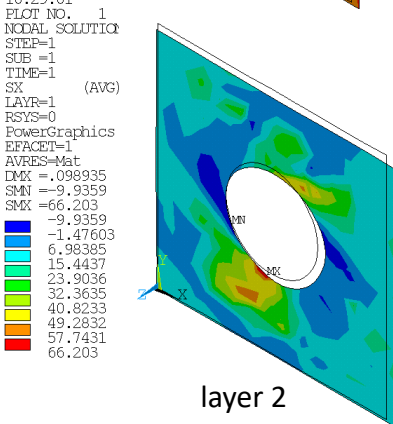
DOF Solution



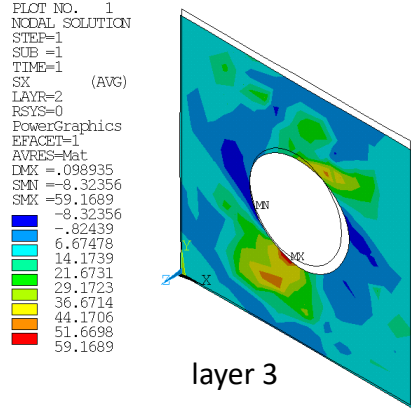
Stress SX [MPa]



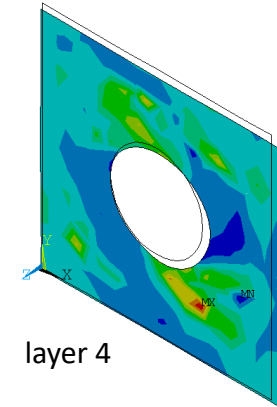
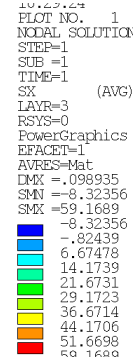
layer 1



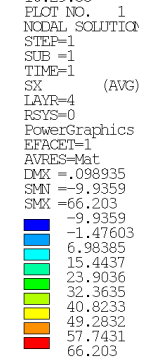
layer 2



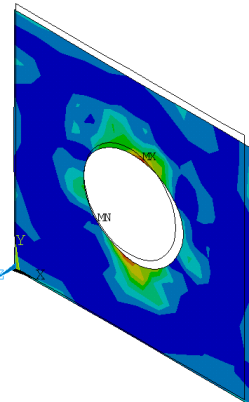
layer 3



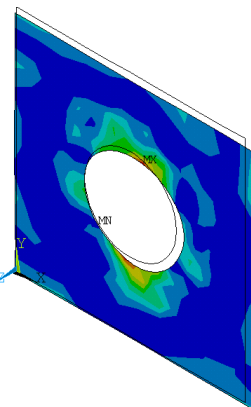
layer 4



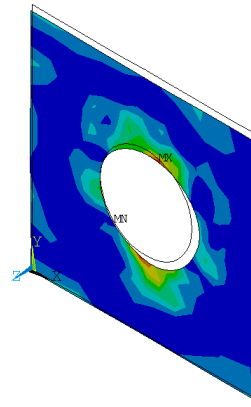
Maximum stress criterion



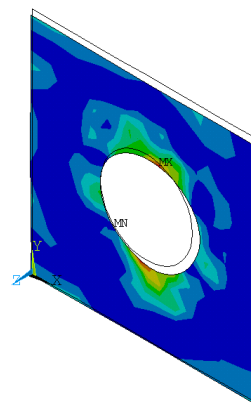
k>1



k>1

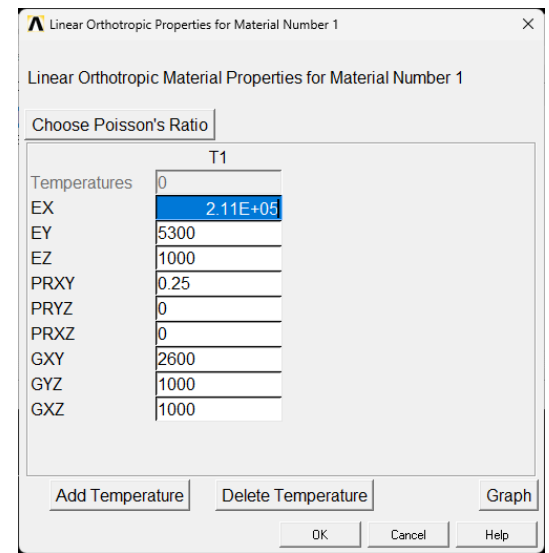
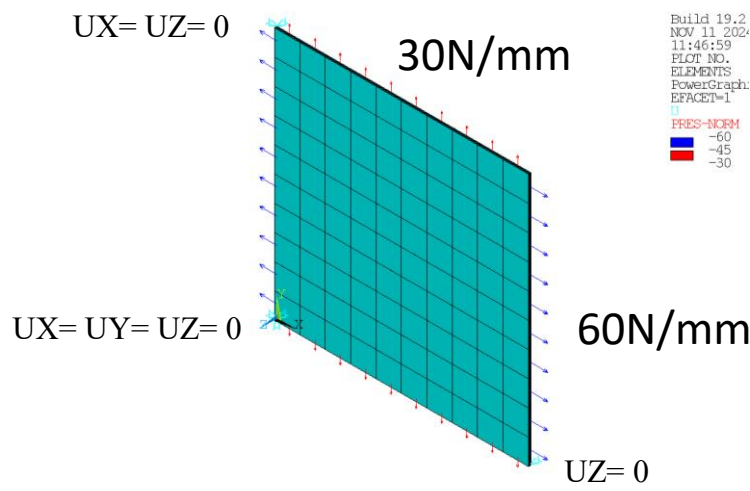


k>1

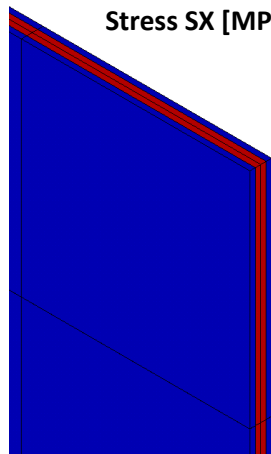
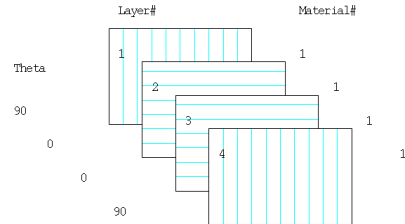


k>1

Example 5. 2D-stretching of a four-layer plate (from Ex. 3)

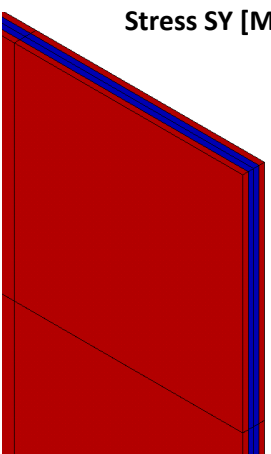


[90°,0°,0°,90°] - symmetric



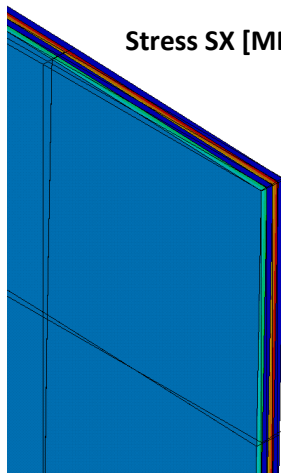
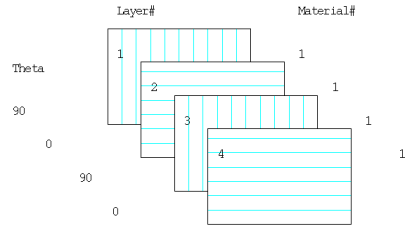
STEP=1
SUB =1
TIME=1
SX (AVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.061333
SMN =3.28138
SMX =116.719
3.28138
60
116.719

Stress SY [MPa]



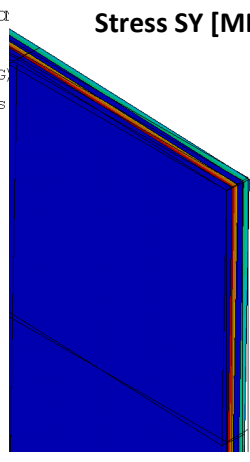
ANSYS Release 1
Build 19.2
NOV 11 2024
11:50:08
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SY (AVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.061333
SMN =2.16507
SMX =57.8349
2.16507
30
57.8349

[90°,0°,90°,0°] - asymmetric



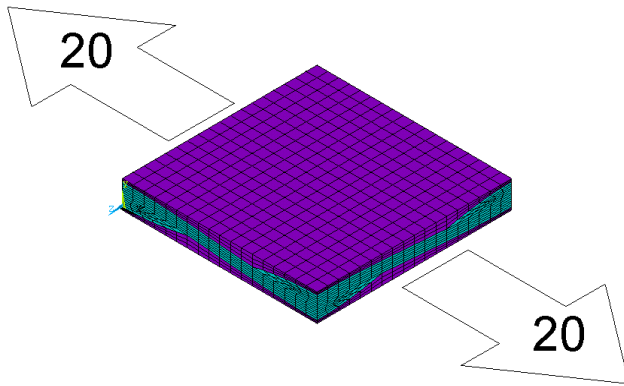
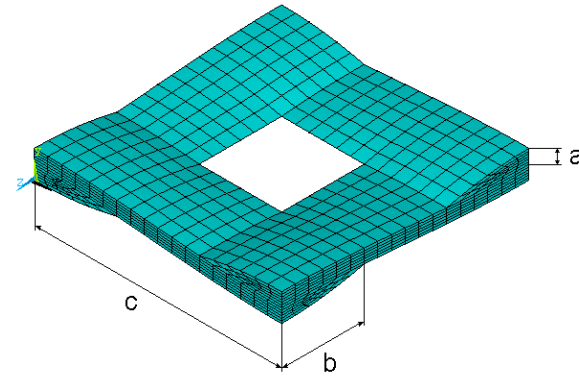
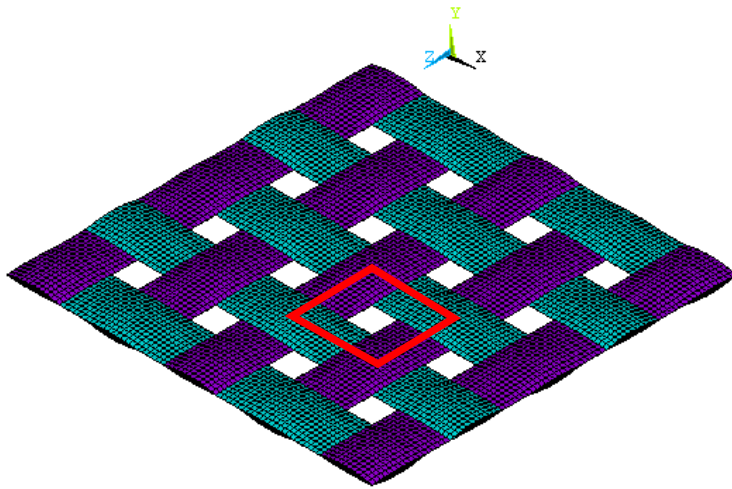
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SX (AVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =1.19101
SMN =2.84648
SMX =190.679
2.84648
23.7167
44.587
65.4572
86.3275
107.198
128.068
148.938
169.809
190.679

Stress SY [MPa]

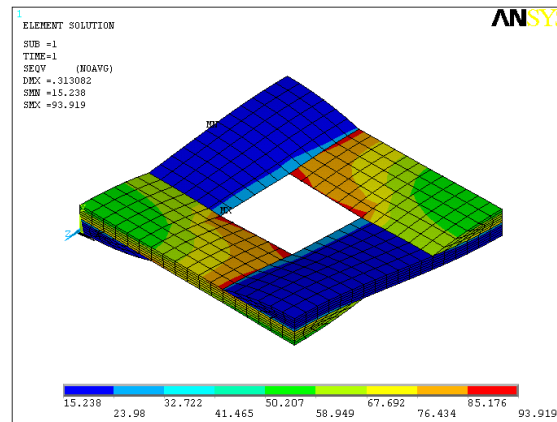


ANSYS Release 1
Build 19.2
NOV 11 2024
11:55:26
PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SY (AVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =1.19101
SMN =2.29152
SMX =94.4711
2.29152
12.5337
22.7759
33.018
43.2602
53.5024
63.7446
73.9867
84.2289
94.4711

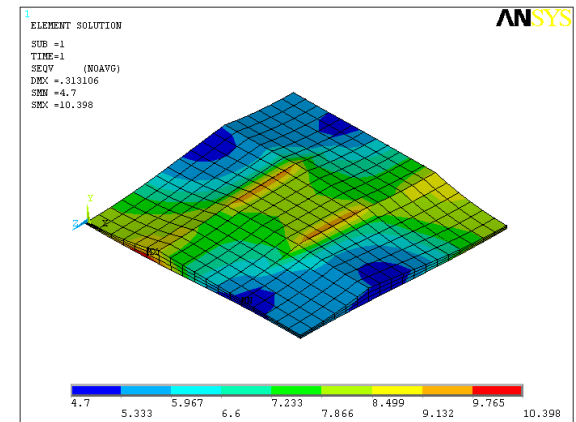
Example 6. Detailed modeling of a single layer (homogenization).



RVE- representative volume element



Von Mises stress in fibers [MPa]



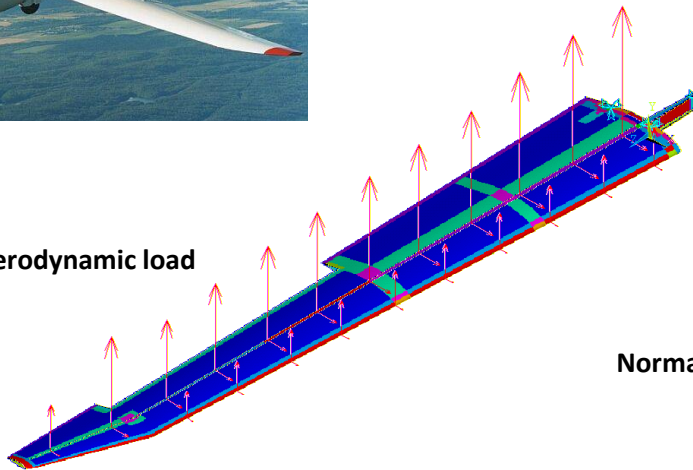
Von Mises stress in matrix [MPa]

Homogenization replaces a complex heterogeneous model with a homogeneous material having substitute material properties.

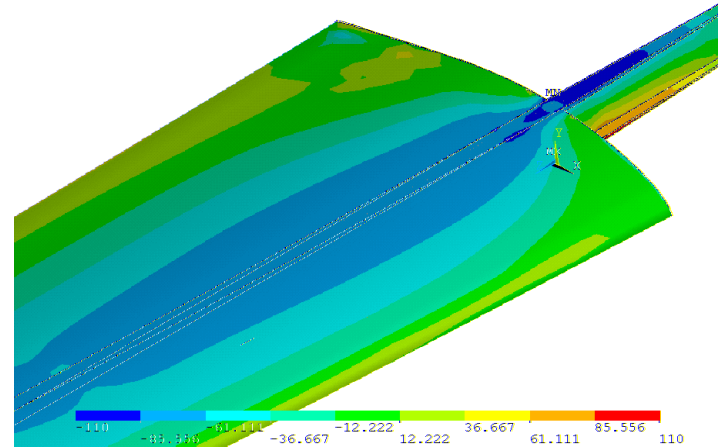
Example 7. FEM analysis of the PW-5 glider wing



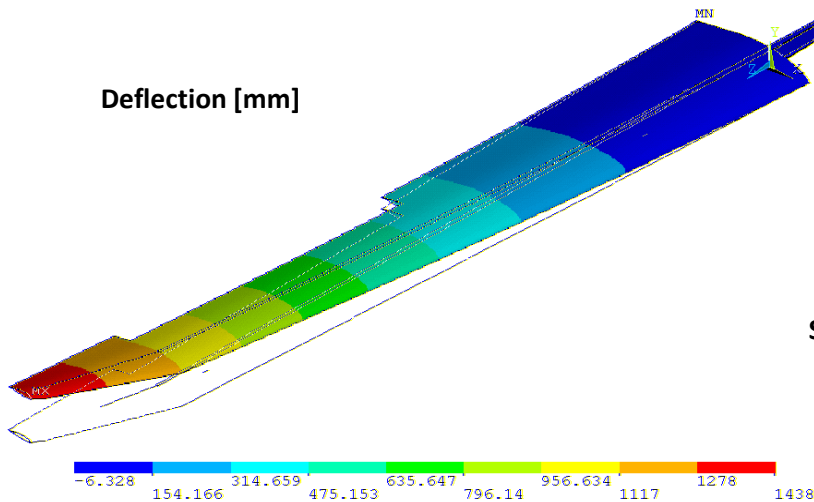
Aerodynamic load



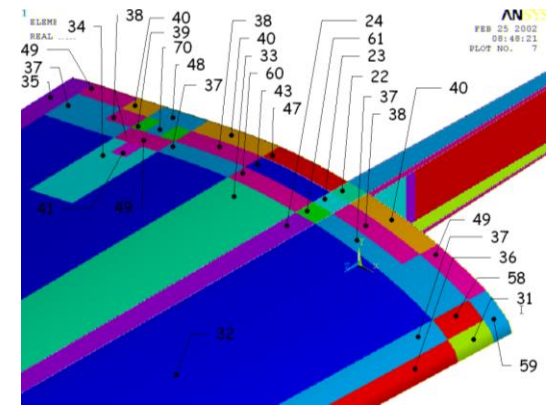
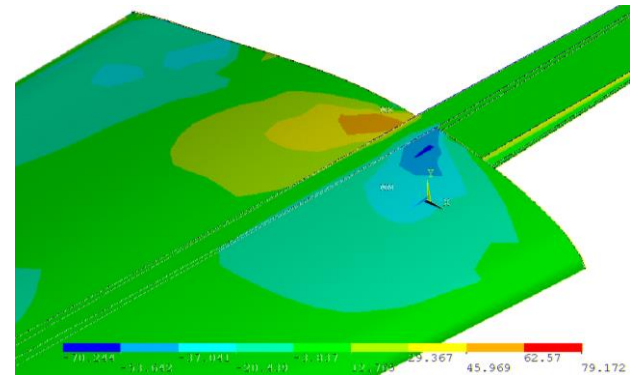
Normal stress [MPa]



Deflection [mm]



Shear stress [MPa]



Reinforcement distribution